UNIVERSITY OF DAR ES SALAAM

THE 2ND AFRICAN–NORDIC CONFERENCE IN MATHEMATICS

BOOK OF ABSTRACTS

THEME

COLLABORATIVE RESEARCH IN MATHEMATICS AND MATHEMATICS EDUCATION FOR SUSTAINABLE DEVELOPMENT

11–14 August 2026

LUSH GARDEN HOTEL, ARUSHA, TANZANIA

MATH 4 SDG

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FOREWORD

Foreword

It is my great pleasure, on behalf of the Local Organizing Committee, to welcome you to the *Book of Abstracts* of the **2nd African–Nordic Conference in Mathematics**, hosted by the University of Dar es Salaam in Arusha, Tanzania, from 11–14 August 2026. This conference brings together researchers, educators, practitioners, and students from Africa, the Nordic countries, and beyond to exchange knowledge, share research findings, and strengthen collaborations in mathematics, mathematics education, and related disciplines.

This *Book of Abstracts* presents the keynote, plenary, and contributed abstracts to be presented during the conference. The contributions span a broad range of topics, including pure and applied mathematics, mathematical modelling, optimisation, biomathematics, statistics, data science, artificial intelligence, financial mathematics, and mathematics education. Together, they demonstrate the breadth of contemporary mathematical research and its growing contribution to scientific innovation, education, industry, and sustainable development.

The abstracts reflect the conference theme, *"Collaborative Research in Mathematics and Mathematics Education for Sustainable Development."* They showcase innovative research, emerging ideas, and interdisciplinary approaches while highlighting the importance of collaboration among researchers, institutions, and countries in addressing local, regional, and global challenges. I hope that this *Book of Abstracts* will serve not only as a guide to the conference programme but also as a valuable reference that will continue to inspire research and collaboration beyond the conference.

The conference is organised under the **Mathematics for Sustainable Development (MATH4SDG) Project**, a partnership between the University of Bergen (Norway), the University of Dar es Salaam (Tanzania), and Makerere University (Uganda). The Organizing Committee gratefully acknowledges the generous support of the MATH4SDG Project, funded by Norad under the NORHED II Programme. I also sincerely thank the **Cluster of Research Excellence in Mathematics (CoRE-Math)** and the **Swedish International Development Cooperation Agency (Sida)** for supporting researchers' participation, as well as the **Inter-University Council for East Africa (IUCEA)** for providing travel grants to participants from member universities across East Africa. These valuable contributions have broadened participation, enriched scholarly dialogue, and strengthened regional collaboration.

Finally, I extend my sincere appreciation to all authors, keynote and plenary speakers, members of the Scientific Committee, reviewers, session chairs, and the Local Organizing Committee for their invaluable contributions and commitment. On behalf of the Local Organizing Committee and the University of Dar es Salaam, I warmly welcome you to the **2nd African–Nordic Conference in Mathematics** and trust that you will find this *Book of Abstracts* both informative and inspiring.

Prof. Eunice Mureithi
Chairperson, Local Organizing Committee
2nd African–Nordic Conference in Mathematics

LOCAL ORGANIZING COMMITTEE

Prof. Eunice Mureithi
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Dr. Makungu James
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PROGRAMME STRUCTURE

Tuesday, 11 August 2026	Wednesday, 12 August 2026	Thursday, 13 August 2026	Friday, 14 August 2026
09:00–10:10 Opening & Welcome	**08:30–09:10** Plenary Speaker 2: *Prof. Martin Singull*	**08:30–09:10** Plenary Speaker 4: *Prof. Bernard Mutuku Nzimbi*	**08:30–09:10** Plenary Speaker 6: *Prof. Timothy Kamanu*
	09:10–10:30 Contributed Talks	**09:10–10:30** Contributed Talks	**09:10–10:10** **Closing Ceremony**
10:10–10:50 **Health Break**	**10:30–11:00** **Health Break**	**10:30–11:00** **Health Break**	**10:10–10:40** **Health Break**
10:50–11:40 Keynote Speaker *Prof. V. G. Masanja* 11:40–12:40 Contributed Talks	**11:00–13:00** Contributed Talks	**11:00–13:00** Contributed Talks	**10:40–18:00** **Excursion & Departure**
13:00–14:00 **Lunch**	**13:00–14:00** **Lunch**	**12:50–14:00** **Lunch**	
14:00–14:40 Plenary Speaker 1: *Prof. B. D. Søndergaard*	**14:00–14:40** Plenary Speaker 3: *Prof. Montaz Ali*	**14:00–14:40** Plenary Speaker 5: *Prof. Farai Nyabadza*	
14:40–15:40 Contributed Talks	**14:40–15:40** Contributed Talks	**14:40–15:40** Contributed Talks	
15:40–16:00 Coffee Break	**15:40–16:00** Coffee Break	**15:40–16:00** Coffee Break	
16:00–17:00 Contributed Talks	**16:00–17:00** Contributed Talks	**16:00–17:00** Contributed Talks	
	17:30–19:00 **Cocktail Party**	**19:00–21:00** **Conference Dinner**	

Remarks: During Contributed Talks, there will be parallel sessions.

KEYNOTE SPEAKERS

Mathematics Beyond the Blackboard: Modelling Africa's Health, Water, Climate, and Environmental Systems for Sustainable Development

Prof. Verdiana Grace Masanja

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Nelson Mandela African Institution of Science and Technology (NM-AIST), Tanzania

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Collaborators and Affiliations

Members of the Mathematical Modelling, Computational Science and Engineering Research Group, NM-AIST, and collaborators from various institutions in Tanzania, Uganda, Kenya, Rwanda, Mozambique, South Africa, Finland, Portugal, and other partner countries.

Abstract

Drawing on a portfolio of over twenty multidisciplinary modelling projects undertaken within the Mathematical Modelling, Computational Science and Engineering Research Group at NM-AIST during the past decade, and which have collectively generated forty-nine peer-reviewed publications, this keynote demonstrates how mathematical modelling provides a unifying framework for addressing sustainability challenges in Africa. The projects integrate deterministic and stochastic differential equation models, optimal control theory, parameter estimation, Bayesian inference, uncertainty quantification, computational fluid dynamics, deep learning, and physics-informed neural networks. These approaches have been applied to infectious disease epidemiology, water distribution and sewer systems, groundwater resources, environmental pollution, climate extremes, and agricultural systems. Findings show how rigorous mathematical analysis coupled with data-driven computation can improve prediction and forecasting, quantify uncertainty, identify critical system drivers, and evaluate intervention strategies. The keynote highlights the growing synergy between mathematical modelling, scientific computing, and artificial intelligence in advancing evidence-based decision making for sustainable development in Africa.

Keywords

Mathematical Modelling; Scientific Computing; Artificial Intelligence; Sustainability; Decision Support

Category

Mathematics for Sustainable Development

References

[1] Charles, M., Mfinanga, S.G., Lyakurwa, G.A., Torres, D.F.M., and Masanja, V.G. (2025). A mathematical and optimal control model for rabies transmission dynamics among humans and dogs with environmental effects. *Journal of Applied Mathematics*, 2025, Article ID 5559842.

[2] Mushumbusi, P.F., Leo, J., Chaudhari, A., and Masanja, V.G. (2025). Modelling and analysis of hydraulic transients in water pipelines using physics-informed neural network. *Scientific African*, 28, e02637.

[3] Kyojo, E.A., Mirau, S., Osima, S.E., and Masanja, V.G. (2024). Modeling non-stationarity in extreme rainfall data and implications for climate adaptation: A case study from southern highlands region of Tanzania. *Scientific African*, 25, e02321.

Reimagining the Mathematics Lecture: Preserving Rigor While Engaging Students Actively

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Department of Mathematical Sciences, Aalborg University, Aalborg, Denmark.

Abstract

This keynote paper explores the overarching question of how learning theories in mathematics can inform the teaching of mathematics at the secondary and tertiary levels. Research grounded in both constructivist and social constructivist theories suggests that learning occurs when students are active participants. The keynote discusses what active participation actually entails and argues that it is not necessarily opposed to traditional lectures. Instead, it raises questions about the types of lectures that genuinely benefit students, the circumstances under which lectures are appropriate, and the roles of both students and lecturers in making lectures effective. Using practical examples, the keynote also illustrates how lectures can be complemented by student-driven approaches such as problem-based and project-based learning. These learning activities may be undertaken by student teams before, during, or after lectures and require effective facilitation by supervisors.

Keywords

Active learning; lectures; problem-based learning; project-based learning; mathematics education.

Category

Mathematics Education

References

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2. Andrade, M. D. O., Dahl, B., & Wolff, K. (2025). Same but not the same: Describing and shaping meaning-making in engineering mathematics problems. In J. Chen, L. Smith, Y. B. Abouarabi, K. Wolff, Z. Simpson, & A. Guerra (Eds.), *10th International Research Symposium on Problem-Based Learning: Anchoring Conversations* (pp. 246–258). Aalborg University Open Publishing.

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6. Gaanya, I., Batiibwe, M. S. K., Dahl, B., Mango, J. M., & Mayende, G. (2025). Mathematics for Sustainable Development implementation at secondary school level: A scoping review. *European Journal of Sustainable Development Research*, 9(4), Article em0321.

7. Kihaga, H., Dahl, B., Kitta, S., & Likinjiye, M. (2025). Strategies for teaching and assessing students in large mathematics classes in secondary schools in Tanzania: A systematic review. *Social Sciences and Humanities Open*, 12, Article 101744.

8. Ssali, C., Batiibwe, M. S. K., Dahl, B., Kaliisa, H., Mango, J. M., & Mayende, G. (2025). Problem-based learning in secondary school mathematics: A review. *Educational Research*, 67(2), 212–230.

Data-driven Model Selection for Bilinear Regression using Functional Principal Component Analysis

Martin Singull

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Abstract

In this presentation we investigate data-driven basis selection in bilinear regression models for functional data, where observations are treated as smooth curves. By combining Functional Data Analysis (FDA) with Functional Principal Component Analysis (FPCA), we construct low-dimensional, interpretable representations of repeated measurements. We compare fixed bases (monomial) with adaptive FPCA-based bases. Using simulations and a benchmark dataset of Canadian temperature curves, we show that FPCA-based representations improve predictive performance, enable efficient dimension reduction, and enhance interpretability. The results underscore the critical role of basis selection and highlight the benefits of integrating FDA, FPCA,

and bilinear regression for functional data modeling. The approach is broadly applicable to data supporting the UN Sustainable Development Goals (SDGs), for example in environmental monitoring and climate-related analyses involving complex functional observations.

Keywords

Data-driven models; Bilinear Regression; Functional data analysis

Category: Statistics

Network-based Data Mining via Matrix Decomposition

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Collaborators and Affiliations

Salma Omer, School of Computer Science and Applied Mathematics, University of the Witwatersrand, Johannesburg, South Africa.

Abstract

We consider the problem of identifying a maximum clique and a maximum edge biclique in given graphs. We propose mathematical models for these problems based on the matrix decomposition of the adjacency matrix of a given graph. The objective function of the proposed model incorporates a weighted L_1-norm of the sparse matrix in the decomposition, providing an advantage over the standard L_1-norm in reducing the error. We employ proximal operators within the iterates of the alternating direction method of multipliers (ADMM) algorithm to solve the resulting optimization problem, and the convergence of the proposed ADMM algorithm is established. We also provide a theoretical guarantee for identifying the maximum clique through the corresponding low-rank matrix. In addition, we propose a natural condition for identifying the maximum edge biclique in random graphs. Numerical results for planted cliques and bicliques demonstrate clear advantages of the proposed model compared with two recent mathematical models. Results for randomly generated graphs show minimal errors, and applications to real-world graphs successfully recover maximum cliques.

Keywords

Maximum clique; maximum edge biclique; matrix decomposition; ADMM; graph mining; optimization.

Category

Graph Theory

Cracking the Invariant Subspace Problem (ISP): Reducibility, Hyper-reducibility, Reductivity, Invariant Subspace Lattices, and Applications

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Collaborators and Affiliations

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Stanley Kathurima Imagiri, Department of Mathematics, University of Nairobi, Nairobi, Kenya.

Abstract

The Invariant Subspace Problem (ISP) remains one of the most enduring challenges in modern operator theory, asking whether every bounded linear operator on a complex Hilbert space possesses a non-trivial closed invariant subspace. This paper systematically explores the geometric and algebraic framework underlying solutions to the ISP through the concepts of reducibility and hyper-reducibility. By examining the topology of invariant subspace lattices, reducing subspaces, and hyperinvariant subspaces, we analyse structural configurations under which non-trivial invariant subspaces are guaranteed to exist. Furthermore, we connect the abstract theory of invariant subspaces with applications in mathematics, physics, data science, and machine learning.

Keywords

Invariant subspace; reducing subspace; hyperinvariant subspace; hyper-reducing subspace; commutant.

Category

Functional Analysis / Operator Theory

References

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2. Bercovici, H., Foias, C., & Pearcy, C. (2008). *On the hyperinvariant subspace problem IV*. Canadian Journal of Mathematics, 60, 758–789.

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Digidemiology: An Epidemiological Framework for Modelling Digital Pathogens

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Abstract

The rapid, unchecked proliferation of harmful digital content across globally networked populations is among the defining public-health challenges of the twenty-first century. Over the past two decades, digital transformation has reshaped epidemiological thinking, giving rise to digital epidemiology, which leverages digital data to surveil biological disease, and infodemiology, which studies how information shapes health outcomes during crises. Yet neither addresses harmful digital entities, from algorithmic manipulation to AI-generated misinformation, that now evolve and spread independently of any biological disease, acting on the mind through cognitive distortion, emotional dysregulation, and psychiatric sequelae rather than on the body. To close this gap, we introduce *Digidemiology*, which we define as the study of the transmission, dynamics, effects, and control of harmful digital entities across cyber-social networks. We establish its conceptual and mathematical foundations, defining constructs such as digital pathogens, digital susceptibility, and algorithmic amplification, and mapping classical epidemiological quantities to their digital analogues. We formulate a compartmental model of digital contagion, derive the digital reproduction number via the next-generation matrix, and prove positivity, boundedness, and the existence and stability of equilibria. These results yield behaviourally interpretable thresholds governing whether harmful content persists or dies out, from which we derive candidate intervention levers. We conclude by outlining five grand challenges and a phased research agenda, positioning digidemiology to become, for the digital world, what classical epidemiology is for biological disease: the quantitative science that explains, predicts, and ultimately controls harmful digital contagion.

Keywords

Digidemiology; Digital contagion; Digital reproduction number; Compartmental models; Algorithmic amplification.

Category

Mathematical Epidemiology (Biomathematics)

ENGAGE: Unlocking AI Talent for Cutting-Edge Transdisciplinary Research in Africa, from Africa

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Abstract

The talk will focus on lessons learned from implementing a data science curriculum in the Kenyan context and beyond. It will draw on experiences across the full process, from conceptualization and formalization to implementation and simulation, with an emphasis on problem-based learning as a pathway to creating and scaling impact within local communities and beyond.

Keywords

Category

Data Science, AI

PURE MATHEMATICS

An Algorithm for the Number of Decomposition Factors of Certain D-Modules

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Abstract

Let $\alpha_1, \ldots, \alpha_m$ be linear forms defined on the vector space $\mathbb{C}^n$, and let $X = \mathbb{C}^n \setminus V(\alpha)$, where $\alpha = \prod_{i=1}^{m} \alpha_i$ and $V(\alpha) = \{p \in \mathbb{C}^n : \alpha(p) = 0\}$. The coordinate ring $\mathcal{O}_X = \mathbb{C}[x]_\alpha$ of X is a holonomic A_n-module, where A_n is the nth Weyl algebra and since holonomic A_n-modules have finite length, $\mathcal{O}_X$ also has finite length. We consider a twisted variant of this A_n-module, which is also holonomic. Define M_α^β to be the free rank-one $\mathbb{C}[x]_\alpha$-module generated by α^β (thought as a multivalued function), where $\alpha^\beta = \alpha_1^{\beta_1} \cdots \alpha_m^{\beta_m}$, and the multi-index $\beta = (\beta_1, \ldots, \beta_m) \in \mathbb{C}^m$.

The main objective of this presentation is to find the decomposition factors of D-modules. This can be solved by developing an algorithm that determines the decomposition factors of the given module. Our main result is developing an algorithm to determine the number of decomposition factors of the module M_α^β.

Keywords

D-modules; decomposition factors; quotient of D-modules; algorithm.

Category

Algebra

References

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An Introduction to Local Isometric Embeddings

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Collaborators and Affiliations

1. Simphiwe Mathenjwa, University of KwaZulu-Natal, South Africa.

2. Sunil Maharaj, University of KwaZulu-Natal, South Africa.

3. Noeleen Naidoo, University of KwaZulu-Natal, South Africa.

Abstract

We provide a survey of both existence results and explicit solutions for local isometric embeddings into higher-dimensional Euclidean and more general pseudo-Riemannian geometries, and generate several new explicit embeddings. Of particular interest are recent results establishing that, for analytical spacetimes, the Euclidean co-dimension is at most two. This casts new light on the Karmarkar condition for class one pseudo-Euclidean embeddings and its analogues for class two embeddings into pseudo-Euclidean spaces and more general embedding spaces, from which we derive a new embedding condition.

Keywords

Embeddings; geometry; gravity; exact solutions.

Category

Mathematical Physics

References

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Progress on Lomonosov-Type Approaches to the Invariant Subspace Problem

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Abstract

The Invariant Subspace Problem (ISP) is one of the most long-standing questions in functional analysis. In its original form the problem asks whether every bounded linear operator T on a complex Banach space X posseses a non-trivial, closed, proper subspace M ((i.e., $M \neq \{0\}, X$) such that $T(M) \subseteq M$. Such a subspace is called an invariant subspace under T. The general problem is now solved in the negative for general Banach spaces (by Per Enflo and Charles Read in the 1980s). For finite dimensional Banach spaces and non-separable Hilbert spaces the answer to the problem is positive. But for separable Hilbert spaces and reflexive Banach spaces the problem is still open. In its present form, the main ISP problem asks whether every bounded linear operator T on a separable complex Hilbert space X possesses a non-trivial, closed, proper subspace M such that $T(M) \subseteq M$. In addition to the search for solution to this main version of the ISP, research directions has shifted towards exploring several related questions, including the existence of non-trivial invariant subspaces for specific operators (e.g., subnormal, weighted shifts), the structure of the invariant subspace lattice Lat(T) and the existence of almost-invariant subspaces and etc. Studying the ISP is crucial for reducing complex operators into simpler forms, providing deep insights into operator dynamics, spectral theory, and applications in quantum mechanics. Fixed-point theorems are one of the various techniques used in studying the invariant subspace theory of operators. A major breakthrough in this approach is Lomonosov's Theorem (1973), which states that if a non-zero compact operator K commutes with T(i.e., $TK = KT$), then T has a non-trivial invariant subspace. This talk will provide an overview of progresses on the so called Lomonosov-type Approaches to the Invariant Subspace Problem.

Keywords

Invariant Subspace Problem; Fixed-Point Theorems; Banach Space; Lomonosov's Theorem

Category

Functional Analysis

References

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Two Parameter Gamma Function and Its Properties

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Abstract

In this paper, we introduce the two-parameter Gamma function, Beta function, and Pochhammer symbol. We name them the p-k Gamma function, p-k Beta function, and p-k Pochhammer symbol, respectively. We prove several identities satisfied by these functions that are analogous to those of the classical Gamma function, Beta function, and Pochhammer symbol. We also provide integral representations for the p-k Gamma function and the p-k Beta function.

Keywords

Two-parameter Pochhammer symbol; two-parameter Gamma function; two-parameter Beta function; two-parameter Psi function; p-k hypergeometric function.

Category

Special Functions

References

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On a Class of Skew Dyck Paths

Dr. Yvonne W. Kariuki

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Collaborators and Affiliations

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Abstract

This paper introduces the class of skew 2-Dyck paths- Dyck-like lattice paths that allow unit up-steps, down-steps of length 2, and left-steps of length 2, provided the paths remain nonintersecting. An explicit enumaration formula for these paths is derived using the symbolic method and the the Lagrange Inversion Formula. In addition, the paper defines three related combinatorial structures: 2-labeled box paths, 3-leaf-labeled plane trees, and 2-edge-labeled plane trees. Bijections are constructed between the set of skew 2-Dyck paths and each of these three structures, thereby demostrating their enumerative equivalence.

Keywords

Box; bijection; binary; log-convex; plane tree.

Category

Combinatorics

References

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The Toeplitz–Hausdorff and Spectral Inclusion Theorems for Linear Relations

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Collaborators and Affiliations

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Abstract

The Toeplitz–Hausdorff Theorem, established in 1918, asserts that the numerical range of an operator is always convex. We prove this theorem for the numerical range of a linear relation relative to another linear relation. We also show that the closure of this numerical range contains the relative spectrum of these two linear relations. The classical results for a single linear relation can be deduced from the results obtained here.

Keywords

Linear relation; numerical range; convexity; spectral inclusion.

Category

Functional Analysis

References

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Algebraic Method of Group Classification for Semi-Normalized Classes of Differential Equations

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Abstract

The theory of semi-normalized classes of differential equations is studied and generalized by using the extended algebraic method of group classification of such classes. We prove the theorems on factoring out symmetry groups and invariance algebras of systems from semi-normalized classes and on splitting such groups and algebras within disjointedly semi-normalized classes. To illustrate the efficiency of the developed theory, we apply it to the group classification problem for the class of (1+n)-dimensional linear Schrödinger equations with complex-valued potentials for the general value of $n \geqslant 1$.

Keywords

Classes of differential equations; Algebraic method; Group classification; Semi-normalization; Schrödinger equations

Category

Algebra

References

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Solvability of Infinite Systems of Third-Order Differential Equations in the Sequence Space c Using Meir-Keeler Condensing Operators

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Collaborators and Affiliations

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3. Sylvester Rugeihyamu, University of Dar es Salaam, Dar es Salaam, Tanzania.

Abstract

In this paper, we present an existence result for ω-periodic solutions of an infinite system of third-order differential equations in the classical Banach sequence space c. The result is obtained using techniques related to measures of noncompactness together with the concept of Meir-Keeler condensing operators, while considering the corresponding Green's function associated with the system. To demonstrate the applicability and significance of the obtained results, several concrete examples are provided.

The research problem is to determine sufficient conditions for the existence of ω-periodic solutions to an infinite system of third-order differential equations in the Banach sequence space c. The study is theoretical and employs measures of noncompactness, Meir-Keeler condensing operators, and Green's function techniques.

Keywords

Infinite systems of third-order differential equations; measures of noncompactness; Meir-Keeler condensing operator; Darbo's fixed point theorem; Green's function.

Category

Differential Equations and Dynamical Systems

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A Categorical Perspective on Gluing

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Abstract

This work examines the concept of gluing in algebraic geometry, with particular emphasis on elucidating its categorical aspects. We construct the categorical framework featuring two central constructs: the gluing category and the gluing functor, under the minimal assumption that the underlying category admits pushouts. Our primary contribution lies in the comprehensive exploration of the gluing property across diverse categories such as topological spaces, (pre)sheaves, and schemes. This framework not only provides a unified methodology applicable to aforementioned categories, but also paves the way for potential extensions to new categories. In this talk, we shall focus on the category of topological spaces and on (pre)sheaves on sites.

Keywords

Gluing, topological spaces, refinement, category theory, functor, limit, Grothendieck sites, (pre)sheaves.

Category

Topology and Geometry

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Extension of Riesz's Representation Theorem to the Set of Interval-Valued Functions

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Abstract

In 1909, F. Riesz stated and proved a fundamental result showing that every bounded linear functional from the set $C[0,1]$ of continuous functions on $[0,1]$, endowed with the supremum norm, to the set of real numbers can be expressed as a Riemann–Stieltjes integral on the interval $[0,1]$ (Kreyszig 1991). This result is now known as Riesz's representation theorem.

The above result has been generalized to various classes $C(X)$ of continuous functions on topological spaces X. Moreover, it has been generalized to the set of p-integrable real-valued functions defined on X.

In this work, we generalize Riesz's representation theorem by replacing a bounded linear functional from the set of continuous functions to the set of real numbers with a bounded convex quasilinear functional defined on the set of bounded Hausdorff continuous functions taking values in the set of closed bounded intervals of real numbers (Anguelov et al. 2006, Anguelov et al. 2009). Furthermore, the Riemann–Stieltjes integral is replaced by the Henstock–Stieltjes integral of interval-valued functions (Ju 2016). Thus, we show that a bounded convex quasilinear functional defined on the set of bounded Hausdorff continuous interval-valued functions on $[0, 1]$ can be represented as a Henstock–Stieltjes integral of interval-valued functions defined on $[0, 1]$.

Keywords

Riesz's representation theorem; Hausdorff continuous functions; Henstock–Stieltjes integral.

Category

Real Analysis

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On the Local Stability of a Certain Class of Polynomial Differential Systems

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Abstract

We state and prove a condition for the local stability of a certain class of two-dimensional systems of polynomial differential equations. We present examples of polynomial differential systems to demonstrate that the local stability condition established for the trivial equilibrium point $(0, 0)$ is sharp and compare our result with the well-known Lyapunov local stability criterion (Lyapunov's second method).

Keywords

Equilibrium; stability; asymptotic stability; differential systems.

Category

Mathematical Analysis (Dynamical Systems)

Some More Sparse Bounds for Rough and Smooth Pseudodifferential Operators

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Abstract

Beltran & Cladek [12] use L^r to L^s bounds to prove sparse form bounds for pseudodifferential operators with Hörmander symbols in $S^m_{\rho,\delta}$ up to, but not including, the sharp end-point in decay m. We further develop their technique, obtaining pointwise sparse bounds for rough pseudodifferential operators that are merely measurable in their spatial variables and an alternative proof of their results which avoids proving geometrically decaying sparse bounds. We also provide sufficient conditions for sparse form bounds to hold and use these to reprove know sparse bounds for pseudodifferential operators with symbols in $S^0_{1,\delta}$ for $\delta < 1$.

Keywords

Pseudodifferential operators; pseudo-pseudodifferential operators; sparse bounds; Littlewood–Paley theory; Hardy–Littlewood maximal function.

Category

Harmonic Analysis

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Distinct Flattened Partitions Avoiding a Pattern of Length Four

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Abstract

Let $\mathcal{P}_n$ denote the set of distinct permutations of length n that arise from the flattening process applied to the partitions of $[n] = \{1, \ldots, n\}$. In this paper, we consider the problem of avoiding a single classical pattern of length four by members of $\mathcal{P}_n$. Let $p_n(\tau)$ denote the number of members of $\mathcal{P}_n$ that avoid the pattern τ. We show that $p_n(\tau) = C_{n-1}$ for all $n \geq 1$ for seven patterns of length four, yielding new combinatorial interpretations of the Catalan number sequence. Furthermore, we show that $p_n(\tau)$ corresponds to the binomial transform of the Catalan numbers for three other patterns. To establish our results, we suitably refine the counting sequence $p_n(\tau)$ in each case to obtain a system of functional equations satisfied by the corresponding generating functions. These functional equations may then be solved explicitly leading to a determination of $p_n(\tau)$ in each case.

Keywords

Distinct flattened partition; pattern avoidance; generating function; kernel method.

Category

Combinatorics

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On the Construction of Group Divisible Designs of Block Size Five with Two Groups of Different Sizes for Specific Values of λ_1 and λ_2

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Abstract

Group divisible designs (GDDs) constitute an important class of combinatorial designs with numerous applications in experimental design, error-correcting codes, and network design. This paper presents a detailed investigation into the construction of group divisible designs with block size five and two groups of unequal sizes. Necessary and sufficient conditions for their existence are derived. Several explicit constructions are developed using difference methods, modular development, and combinatorial configurations, with fully worked examples illustrating the constructions for specific parameter sets. In addition, general construction frameworks and infinite families of designs are proposed, providing further insight for both theoretical and applied combinatorial design.

Keywords

Group divisible designs; Block size five; Unequal groups

Category

Combinatorics

An Extension of Pólya's Shire Theorem

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Abstract

The classical Pólya's Shire Theorem states that, for a meromorphic function f in $\mathbb{C}$ with the set S of its poles, the zeros of its iterated derivatives $f^{(n)}$ asymptotically accumulate, as $n \to \infty$, along the edges of the Voronoi diagram associated with S. In this paper, we extend Pólya's Shire Theorem to differential operators with Laurent monomial coefficients, $\mathcal{D}_{-\ell} := z^{-\ell}\frac{\partial}{\partial z}$, where $\ell = 1, 2, 3, \ldots$, acting iteratively on rational functions with a single pole, $r(z) = \frac{-1}{z+b}$, and to differential operators with monomial coefficients of degree $\ell \geq 3$, $\mathcal{D}_{\ell} := z^{\ell}\frac{\partial}{\partial z}$, acting iteratively on arbitrary rational functions with $k \geq 2$ distinct simple poles of the form $h(z) = \sum_{i=1}^{k} \frac{\alpha_i}{z-z_i}$. The results show that, in both cases, the zero loci of the iterates of each differential operator concentrate on specific parts of the corresponding collections of lemniscates in the complex plane. Moreover, the mapping $w(z) = \frac{z^{1-\ell}}{1-\ell}$, $\ell \geq 3$, sends the limiting set of the zeros of $\mathcal{D}_{\ell}^{n}(h(z))$ homeomorphically onto the edges of the Voronoi diagram in the w-plane.

Keywords

Differential operators; lemniscates; Voronoi diagram; rational functions; Laurent monomial coefficients.

Category

Complex Analysis

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On the Enumeration of Subgroup Chains in the Direct Product $\mathbb{Z}_{p^n} \times A_3$ Using the Annihilator Method

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Collaborators and Affiliations

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Abstract

The enumeration of subgroup chains is a fundamental problem in combinatorial group theory with connections to the study of subgroup lattices and fuzzy subgroups. This problem can be formulated combinatorially on the subgroup lattice of a group G: determining the number of chains of subgroups terminating in G (see [1], [2]). This paper studies the direct product $\mathbb{Z}_{p^n} \times A_3$, where p is prime and $n \geq 1$. This group provides a model combining the structure of a cyclic p-group with that of the alternating group A_3. Using an enumerative approach based on classifying subgroups by order and isomorphism type, we derive **first-order linear non-homogeneous recurrence relations** with constant coefficients for the number of subgroup chains. These recurrences are solved explicitly using the annihilator method, yielding **closed-form formulas** for the number of subgroup chains. The formulas are expressed as polynomial functions in the prime p and exponent n. This result advances the systematic enumeration of subgroup lattices in classical group theory.

Keywords

Subgroup chains; Alternating groups; Recurrence relations

Category

Group Theory; Fuzzy Group Theory

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Extended Spectrum of Shift–Diagonal Operators on ℓ^p Spaces

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Abstract

This paper investigates the extended spectral properties of a class of composition operators acting on the sequence spaces ℓ^p, where $1 \leq p < \infty$. The operator is defined as the composition of the unilateral right shift and a diagonal multiplication operator generated by a null sequence $\alpha = (\alpha_k)_{k\in\mathbb{N}}$, given by $C_\alpha x = (0, \alpha_1 x_1, \alpha_2 x_2, \ldots)$. Using norm estimates, spectral radius arguments, and explicit constructions of intertwining operators, we show that the point spectrum, approximate point spectrum, and spectrum of C_α all coincide and reduce to the singleton set $\{0\}$, rendering the operator quasinilpotent. In contrast, we prove that the extended spectrum coincides with the complement of the open unit disc in the complex plane. A comparison with the unilateral shift and multiplication operators demonstrates that the extended spectral properties are highly sensitive to operator composition. These results highlight the sharp divergence between classical and extended spectral theories and contribute to the study of extended eigenvalues on Banach sequence spaces.

Keywords

Extended spectrum; Shift–diagonal operators; Composition operators; ℓ^p spaces

Category

Functional Analysis

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A Note on Dendrite and Terminal Decomposition

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Abstract

This paper presents some notions on dendrites and investigates the relationships between topological transitivity, mixing, and periodic points on dendrites in view of the results presented in [1]. The main result shows that a terminal periodic decomposition exists for topological maps. In particular, we show that there exists a regular closed decomposition $D = D_0, \ldots, D_{m-1}$ for the dendrite D with finite decomposition ideal $DI(T) = \{1\}$.

The research problem arises from the results in [1], where it remains unknown whether every transitive dynamical system on dendrites admits a terminal periodic decomposition. The study is purely theoretical and is based on an extensive review of the existing literature.

Keywords

Dendrite; terminal periodic decomposition; decomposition ideal; periodic point; ω-limit set.

Category

Ergodic Theory and Dynamical Systems

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Mann-type composite iterative method for a generalized equilibrium problem, constrained convex minimization problem and fixed point problem of a nonlinear mapping

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Abstract

We introduce a new Mann-type composite iterative algorithm to approximate a common solution of a generalized equilibrium problem, a constrained convex minimization problem, a minimization of quadratic function, and fixed point problem of a family of nonlinear mappings in a Hilbert space setting. We prove strong convergence of the proposed iterative scheme to a common solution that satisfies a certain variational inequality problem. The results improve and extend the approximation schemes in solving combination of the mentioned problems introduced by some authors in the literature.

Keywords

η-relaxed monotone mapping, fixed points, variational inclusion problem.

Category

Fixed Point Theory / Nonlinear Analysis

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Solvability of Infinite Systems of Integral Equations of Hammerstein Type in Three Variables in Tempering Sequence Spaces c_0^β and ℓ_1^β

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Abstract

The objective of this study is to determine the criteria under which an infinite system of Hammerstein-type integral equations in three variables admits a solution in the Banach tempering sequence spaces c_0^β and ℓ_1^β, using Meir–Keeler condensing operators. Our research builds upon the findings of Malik and Jalal [**?**]. Furthermore, we provide illustrative examples to demonstrate the applicability of the established conditions.

Keywords

Hausdorff measure of noncompactness; Meir–Keeler condensing operators; systems of integral equations; tempering sequence spaces.

Category

Functional Analysis

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On Existence of Subspace Codiskcyclicity for C_0-Semigroups

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Abstract

This study introduces the concept of subspace codiskcyclicity for C_0-semigroups on infinite-dimensional separable Banach spaces, extending the existing notions of diskcyclicity and codiskcyclicity. The research problem is to determine whether a C_0-semigroup can exhibit codiskcyclic dynamics relative to a nontrivial closed subspace, even when it is not codiskcyclic on the whole space. Using methods from linear dynamics and operator theory, we establish a subspace codiskcyclicity criterion that provides sufficient conditions for such behavior. Key findings include: (i) every codiskcyclic C_0-semigroup is automatically subspace codiskcyclic with respect to some nontrivial subspace; (ii) if a single operator in the semigroup is subspace codiskcyclic, then the whole semigroup inherits this property; (iii) subspace codisk topological transitivity is equivalent to subspace codiskcyclicity. We also construct concrete examples. The conclusion is that subspace codiskcyclicity forms a strictly larger class than codiskcyclicity, enriching the theory of linear dynamical systems.

Keywords

Subspace codiskcyclicity; C_0-semigroups; codiskcyclic operators; hypercyclicity; linear dynamics

Category: Functional Analysis / Operator Theory

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An Efficient Miniball Approximation Approach for Estimating Simplicial Complexity in a Stable Bifiltrations

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Abstract

This study explores the computational complexity of stable bifiltrations for point cloud(s), with a particular focus on multicover bifiltrations. Bifiltrations extend the classical notion of persistent homology from challenges of data with high-dimension, large, noisy and outliers by capturing topological features across two parameters, offering a richer framework for analyzing complex data.

The research is driven by developing an efficient algorithm for their computation called miniball algorithm, and incoperate to analyze the computational complexity associated with constructing these bifiltrations, particularly in terms of the number of simplices required using resolution and dimension constraints.

The study addresses key questions around algorithmic efficiency and topological accuracy, aiming to provide new strategies of miniball algorithm for simplifying and accelerating computations in topological data analysis (TDA) based on the above three objectives.

The outcomes of this work are significant not only for advancing theoretical understanding in computational topology, but also for enabling practical applications across various scientific disciplines that involve the analysis of large and high-dimensional datasets.

Keywords

Filtration; Bifiltration; Miniball; homology; Nerve

Category

Algebraic topology

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APPLIED MATHEMATICS & MATHEMATICAL MODELLING

Advanced Krylov–Newton Iterative Solvers for the Yang–Baxter Matrix Equation

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Abstract

The Yang–Baxter matrix equation plays an important role in quantum information theory, nonlinear control, and scientific computing. However, its nonlinear structure and large-scale nature make the computation of accurate numerical solutions computationally challenging. In this study, advanced iterative techniques for solving the Yang–Baxter matrix equation were investigated with emphasis on Krylov–Newton techniques, accelerated fixed-point iterations, and low-rank projection methods. The developed approaches achieved faster convergence, improved numerical stability, and reduced computational cost compared with classical iterative schemes. Theoretical analysis and numerical experiments showed that Krylov-based iterative solvers significantly reduced memory requirements while preserving high approximation accuracy for large-scale problems. The results demonstrate that modern iterative solvers provide efficient and scalable computational tools for solving nonlinear matrix problems arising in contemporary scientific and engineering applications.

Keywords

Yang–Baxter matrix equation; Krylov–Newton methods; low-rank iterative methods; nonlinear matrix equations; numerical linear algebra

Category

Numerical Linear Algebra

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Effects of Viscous Heating and Chemical Reactions on Heat and Mass Transfer in Casson Fluid Flow Through a Channel

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Abstract

The impact of viscous heating and chemical reactions on thermal and solutal transport in non-Newtonian fluids remains a significant challenge in engineering and industrial processes. This study investigates the temperature and concentration distributions of a Casson fluid flowing through a channel, with the aim of enhancing heat transfer and controlling chemical reactivity. A two-step exothermic reaction–diffusion mechanism is incorporated into the Casson fluid model to simulate realistic reactive transport.

The analysis assumes negligible reactant consumption, with the flow driven by pressure gradients and influenced by both the fluid's yield stress and an externally applied Lorentz force. The governing partial differential equations are formulated subject to fixed-wall and Navier slip boundary conditions and transformed into a dimensionless system. The resulting equations are solved using the Galerkin weighted residual method to capture the coupled effects of viscous heating, Casson fluid nonlinearity, and chemical reactivity. Results, presented graphically and

in tabular form, indicate that the inclusion of a second-step exothermic reaction significantly enhances the temperature and concentration distributions, thereby promoting more uniform heat and mass transfer. Furthermore, the apparent viscosity of the Casson fluid increases with higher yield stress parameters, leading to more stable flow behaviour. These findings demonstrate that, although viscous heating and reactive mechanisms can improve heat transfer, careful management is required to prevent excessive temperature rise, which may result in thermal runaway or system instability. The study provides theoretical insights applicable to the design of more efficient, safer, and environmentally sustainable thermal and chemical processing systems.

Keywords

Casson fluid; Two-step exothermic reaction; Viscous heating; Navier slip; Thermal stability; Spectral relaxation method

Category

Fluid Dynamics

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Exploring the Convergence of Mathematics and Chemistry: Insights from Chemical Graph Theory and Linear Regression Models

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Abstract

This presentation reports on research at the intersection of graph theory and chemistry, highlighting their relationship in modelling molecular structures. Drawing on collaborative research with scholars from various disciplines, it provides an overview of work in chemical graph theory and outlines a general methodology for conducting investigations in this field. As a case study, the presentation discusses K-Banhatti indices and entropy measures for rhodium(III) chloride ($\mathrm{RhCl_3}$) using linear regression models, illustrating the application of graph-theoretical techniques to the analysis of chemical compounds.

Keywords

Chemical graph theory; graph entropy; K-Banhatti index; linear regression; molecular descriptors.

Category

Graph Theory

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A Rational Block Hybrid Method for Solving Third-Order Boundary Value Problems

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Abstract

This study presents a rational block hybrid method for solving third-order boundary value problems arising in fluid dynamics. The proposed approach combines rational block hybrid schemes with quasilinearization techniques to efficiently solve nonlinear Falkner–Skan-type equations. The method employs carefully selected grid points based on rational approximations that minimize local truncation errors and improve convergence properties. Theoretical analysis establishes the order, consistency, zero-stability, convergence, and absolute stability of the proposed scheme. Numerical experiments were conducted on the Blasius, Pohlhausen, and Homann flow problems using a MATLAB implementation. The results demonstrate that increasing the number of grid points within each interval significantly improves accuracy, with errors approaching machine precision for suitable parameter choices. The method also achieves favourable computational efficiency while maintaining high accuracy. The findings show that the proposed rational block hybrid method is a reliable and efficient numerical technique for solving third-order boundary value problems encountered in applied mathematics and fluid mechanics.

Keywords

Boundary value problems; rational block hybrid method; Falkner–Skan equation; numerical analysis; fluid dynamics.

Category

Numerical Analysis

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Mathematical Study Groups with Industry in Ethiopia: Experiences and Lessons from the ENNCoMat Network

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Abstract

Mathematical Study Groups with Industry (MSGI) provide a framework for collaboration between academia, industry, and the public sector through the application of mathematical modelling, simulation, and optimization to real-world challenges. In Ethiopia, this concept has been promoted through the Ethiopian–Norwegian Network in Computational Mathematics (ENNCoMat).

This presentation reviews the MSGI concept and reflects on the experiences of the two Ethiopian Study Groups with Industry organized in 2023 and 2025 within the ENNCoMat programme. The study groups brought together researchers, students, and industrial partners to work collaboratively on practical problems while fostering capacity building and international cooperation.

Key outcomes and lessons learned will be discussed, including the benefits of industry–academia collaboration and the development of industrial mathematics in Ethiopia. An illustrative case study on the mathematical modelling and simulation of rubber curing processes in tyre manufacturing demonstrates how mathematical methods can support process optimization and innovation in industrial production.

Keywords

Study Groups with Industry; mathematical modelling and simulation; rubber curing process.

Category

Industrial and Applied Mathematics

References

Graph-Theoretic Models for Efficient and Sustainable Resource Allocation

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Abstract

This study applies combinatorial and graph-theoretic methods to model sustainable resource allocation systems. Demand centres and supply routes are represented as weighted networks, enabling their analysis using dominating sets, matchings, and shortest-path algorithms [3]. The proposed model incorporates sustainability factors, including distribution cost and environmental impact. Results from simulated scenarios indicate that optimised network structures significantly improve coverage and reduce inefficiencies, with hybrid matching–domination techniques achieving up to a 35% improvement compared with non-optimised networks [2]. Furthermore, the work of Ahuja et al. [1] demonstrates the relevance of combinatorial optimisation in supporting evidence-based decision-making for the distribution of water, energy, and educational resources. The study highlights the important contribution of discrete mathematics to sustainable development initiatives.

Keywords

Combinatorics; Graph theory; Optimisation; Sustainable development; Resource networks

Category

Combinatorics

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Variable-Coefficient Hybrid Schemes for Third-Order Initial Value Problems

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Abstract

Despite the numerous numerical techniques available for integrating third-order differential equations of the form $y'''(x) = f(x, y(x))$ in the literature, little or no attention has been given to the fact that the solutions of most of the equations exhibit oscillatory or exponential behavious or combination of both, which might not be properly handled by the general purpose methods/schemes. To address this, a class of trigonometrically and exponentially fitted hybrid schemes is proposed in this paper to integrate this class of problems directly. All the numerical properties that include accuracy, efficiency and stability are analyzed. The schemes are validated on test problems that possess exact solutions. Comparison of the results obtained with those of the existing schemes in the literature reveals a remarkable improvement on efficiency and accuracy with the proposed schemes.

Keywords

Hybrid schemes; Numerical solution; Oscillatory solutions; Exponential fitting; Trigonometric fitting

Category

Numerical Methods/Analysis

A Novel Dynamic Differential Evolution Optimization Method for Weight Optimization in WASPAS

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Abstract

This paper proposes a Dynamic Differential Evolution Optimization Method for determining criterion weights in the Weighted Aggregated Sum Product Assessment (DDEM-WASPAS) framework. The study is motivated by a practical limitation in many WASPAS applications, namely the reliance on fixed or externally assigned weights that may not adequately reflect the discriminatory structure of the decision data. To address this issue, the proposed method combines criterion-wise differential dispersion with an adaptive evolutionary search and embeds the resulting optimization directly within the WASPAS ranking process.

The empirical part of the study is based on exact benchmark data from two published supplier-selection problems. The first benchmark concerns agri-food green supplier selection, while the second concerns PVC-carpentry supplier selection.

The results show that DDEM-WASPAS preserves the leading supplier across all benchmark cases and exactly reproduces the published ranking in the sunflower-soybean group, while introducing only minor changes among closely competing middle alternatives in the other cases. More importantly, the method consistently improves score separation compared with the existing approaches, indicating greater discriminatory power without disrupting the robust dominance patterns already present in the data. These findings suggest that the proposed approach is best understood as a refinement of WASPAS that strengthens the determination of criterion weights. Overall, the paper demonstrates that adaptive, data-driven weight optimization can enhance the real-world performance of WASPAS in ranking conflicting criteria.

Keywords

WASPAS; differential evolution; criterion-weight optimization; green supplier selection; agri-food supply chain; multi-criteria decision making.

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Operations Research and Optimization

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Adaptive Chaotic Optimization of Physics-Informed Extreme Learning Machines: Conditioning, Stability, and Operator-Theoretic Error Bounds

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Abstract

Physics-informed learning methods may be interpreted as mesh-free discretisations of operator equations, yet their numerical stability is often obscured by optimisation-centred formulations. This work develops a numerical analysis framework for Physics-Informed Extreme Learning Machines (PIELMs) by viewing them as operator-based least-squares discretisations of linear boundary value problems.

Let $L : V \to W'$ and $B : V \to Z'$ define a well-posed boundary value problem in a Hilbert space setting. A PIELM induces the nonlinear trial space

$$V_h = \operatorname{span}\{\sigma(w_j^\top x + b_j)\}_{j=1}^{M} \subset V,$$

with coefficients obtained from the discrete least-squares problem

$$\min_{\beta \in \mathbb{R}^M} \|H_L \beta - y\|_2^2,$$

where H_L is the operator-applied feature matrix. Unlike Galerkin schemes, whose stability follows from coercivity, residual-minimisation solvers inherit stability solely through the spectral properties of H_L. Under standard well-posedness assumptions, perturbation theory yields the amplification bound

$$\frac{\|\delta\beta\|_2}{\|\beta\|_2} \leq \kappa(H_L) \frac{\|\delta y\|_2}{\|y\|_2},$$

establishing the condition number of the discrete operator as the governing quantity for numerical robustness.

To regulate this conditioning, we introduce an Adaptive Chaotic Whale Optimization Algorithm (ACWOA) for hidden-layer parameter selection. Rather than serving as a black-box heuristic,

ACWOA is interpreted as a feature-space conditioning regulator: chaotic initialisation promotes ergodic sampling of the parameter space, while adaptive control preserves the singular value separation of H_L. This yields an implicit regularisation mechanism analogous to basis adaptation in classical numerical methods.

The analysis clarifies the distinction between Galerkin projections and operator-based residual minimisation, separates approximation error from numerical instability, and links feature optimisation to discrete operator theory. Numerical experiments for elliptic model problems confirm that conditioning-aware feature adaptation significantly reduces error amplification and improves convergence smoothness without increasing the model dimension.

The results position PIELMs within the framework of numerical linear algebra, perturbation theory, and operator discretisation, providing a mathematically transparent foundation for physics-informed scientific machine learning.

Keywords

Operator theory; Least-squares discretisation; Conditioning; Perturbation analysis; Physics-informed learning; Extreme learning machines

Category

Numerical Analysis

A Nonlinear ODE Model for Financial System Stability and Debt Dynamics

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Abstract

This study develops a nonlinear ordinary differential equation (ODE) model to examine financial system stability and debt dynamics in Tanzania, extending the foundational framework of Keen's (1995) financial instability hypothesis. The research addresses the problem of rising public debt, recurrent inflationary pressures, and vulnerability to macro-financial shocks that challenge stability in emerging African economies. Using nonlinear dynamical systems methods, including equilibrium analysis, Jacobian-based local stability assessment, and bifurcation detection, the model captures interactions among output, government debt, interest rates, and inflation. Simulation results reveal the presence of multiple equilibria, with stability highly sensitive to borrowing costs and fiscal discipline. Bifurcation analysis shows that small policy

shifts can trigger transitions to instability, characterized by rising debt ratios and output contraction. The findings suggest that coordinated monetary and fiscal interventions are essential for maintaining stability and that proactive management of interest rates and expenditure can prevent the system from entering unstable debt-driven cycles.

Keywords

Financial stability; Nonlinear ODE model; Debt dynamics; Bifurcation analysis; Tanzania economy.

Category

ODEs and Dynamical Systems

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Application of the COPRAS (Complex Proportional Assessment) method in conjunction with the Entropy Weight approach

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Abstract

This study explores the application of the COPRAS (Complex Proportional Assessment) method in conjunction with the Entropy Weight approach to evaluate and prioritize crops for agro-processing in Idamba Ward, Njombe, Tanzania. By combining these advanced methodologies, the research aims to develop effective strategies for optimizing crop selection in agro-processing, thereby boosting agricultural productivity and promoting socio-economic development in the region. The study conducts a detailed analysis of the results, which highlights the superior performance of Avocado compared to other crops such as beans, tea, coffee, and maize, all of which are commonly cultivated in Idamba ward. Avocado is identified as not only a high-performing crop but also a strategically important asset, contributing significantly

to the region's agricultural landscape. The study offers actionable recommendations for the sustainable advancement of agro-processing initiatives in Idamba Ward, Njombe, Tanzania. In addition, it provides a structured framework for resource allocation, guiding agro-processing practices towards both growth and long-term regional development. Through these insights, the research underscores the potential for Avocado to significantly enhance agro-processing outcomes and contribute to the overall socio-economic development of the region. The integration of the COPRAS method and the Entropy Weight approach establishes a robust foundation for informed decision- making in crop selection. This dual-method approach emphasizes the critical role of Avocado in driving sustainable agricultural and economic progress in Idamba Ward. By fostering an environment conducive to optimized crop selection, the study ultimately aims to support the sustainable and prosperous development of the agro-processing sector in the region, ensuring long-lasting impacts on the local community's livelihood and economic stability.
Keywords

Category

Distributed Joint Chance-Constrained Optimization via Conditional-IndependenceDecomposition

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Abstract

Multi-agent optimization with joint chance constraints is challenging because the joint satisfaction probability couples the agents through the law of the disturbance, so even with separable cost and affine physical coupling the resulting problem is generally neither separable nor convex. This paper develops a decomposition that respects, rather than discards, the dependence structure of the uncertainty. We model the correlated disturbance through a latent common variable such that the agents' disturbances become conditionally independent. This is natural in multi-agent systems sharing environmental conditions such as neighboring microgrids sharing weather conditions. Under this structure, the joint chance constraint admits the exact factorization, and Jensen's inequality yields anagent-separable safe surrogate in terms of the log-average conditional reliabilities. The main contributionis a convexity result: when the conditional density is log-concave and the constraint set is convex, the local Jensen constraint defines a convex feasible region, and the full decomposed problemis convex. A case study on multi-prosumer coordination with shared community storage illustrates the framework.

Keywords

Multi-agent system; Joint chance constraint; Jensen's inequality; Conditional independence, Distributedoptimization

Category

Optimazation

Solving an Inverse Heat Conduction Problem with Phase Transitions Using Tikhonov Regularization

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Abstract

In our previous work, we introduced an iterative algorithm for the nonlinear inverse heat conduction problem, in which material parameters may depend on temperature, and demonstrated its effectiveness in practical applications(Ngendahayo et al. 2021). A formal convergence analysis was provided in Ngendahayo Et al. (2025), thereby establishing a solid theoretical foundation for the method. In practical applications, such as heat treatment of steel, phase transitions often occur within the material. In these cases, the temperature dependence of the material parameters cannot be expressed as a simple functional relationship of the form $\kappa = \kappa(u)$. Instead, the full temperature history $u(x,t)$ determines which phase transitions actually occur within the material. Consequently, the material properties at a given point (x,t) depend not only on the temperature $u(x,t)$, but also on the initial state of the material at $t = 0$ and the entire temperature history $u(x,\tau)$ for $0 \leq \tau \leq t$. In this work, we aim to introduce a more precise mathematical description of the nonlinearity caused by the presence of phase transitions in the material. We also extend the convergence analysis in Ngendahayo Et al. (2025) to account for phase transitions. In addition, we present numerical experiments demonstrating that the iterative algorithm performs well under these conditions.

Keywords

Nonlinear; inverse heat conduction problem; phase transitions.

Category

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GIS-Based Extended Maximum Covering Location Model for Optimal Fire Hydrant Allocation under Spatial Uncertainty: A Case Study of Dar es Salaam, Tanzania

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Abstract

Fires pose a major threat to urban safety and infrastructure, particularly in rapidly growing cities where fire hydrant placement is often inefficient and unable to provide adequate emergency coverage. This study proposes a GIS-based extended Maximum Covering Location Problem (MCLP) model for the optimal allocation of fire hydrants in Dar es Salaam, Tanzania. The framework integrates Geographic Information Systems (GIS), network-based accessibility analysis, spatial optimization, and uncertainty in demand and service accessibility to maximize fire-risk coverage under hydrant budget and spacing constraints. Results show that the optimized configuration substantially improves spatial coverage and reduces service gaps compared to unplanned deployment strategies. Sensitivity and robustness analyses further demonstrate that the proposed model maintains stable performance under varying operational conditions and spatial uncertainty. The study concludes that the proposed GIS-integrated optimization framework provides an effective decision-support tool for strategic fire hydrant planning in rapidly expanding urban environments.

Keywords

Fire hydrants; Maximum Covering Location Problem (MCLP); spatial optimization; Geographic Information Systems (GIS); urban fire risk; emergency response; uncertainty.

Category

Optimization

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Spatial-temporal Analysis of a Predator-Prey Model with Harvesting and Intraspecific Competition

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Abstract

This study analyzes a harvested predator-prey system with intraspecific competition among predators in a spatially structured environment. A reaction-diffusion model with Neumann boundary conditions is formulated, incorporating prey harvesting and predator self-limitation. Numerical simulations using the IMEX Euler method revealed that harvesting rate r_1 controls system persistence while a transcritical bifurcation occurs at $r_1 = 1$ beyond which prey collapse leading to extinction. Furthermore, it was observed that moderate harvesting $r_1 < 1$ stabilizes dynamics and enables well-organized spatial patterns (spots and stripes) via Turing instability. Intraspecific competition (a_3) governs pattern quality in which strong competition ($a_3 = 0.9$) produced sharp and persistent spots, while weak competition ($a_3 = 0.1$) led to diffusive and irregular aggregations. These results demonstrated that sustainable harvesting and internal predator regulation jointly determine ecosystem stability and spatial structure.

Keywords

Predator-prey; Harvesting; Intraspecific competition; Spatial patterns; Bifurcation

Category

Biomathematics (applied ecology)

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Personalized PageRank Analysis on Hybrid Graphs: A Line of Nodes Connected with Multiple Complete Graphs

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Abstract

Personalized PageRank (PPR) is widely used to measure node importance in networks, but explicit analytical results for heterogeneous graph structures remain limited. This study investigates PPR on hybrid undirected graphs consisting of a path graph connected to several complete graphs through bridge edges. Using block matrix decomposition, spectral analysis, and perturbation theory, closed-form formulas are derived for PPR values within attached cliques in terms of a single inflow parameter from the path. It is shown that when personalization is concentrated in a clique and the damping factor is sufficiently large, the seeded clique accumulates more PageRank mass than the entire path. A first-order sensitivity formula is also obtained to describe how PPR changes under bridge modifications. Numerical experiments confirm the theoretical results. The study provides an interpretable framework for understanding localization and ranking stability in structured and evolving networks.

Keywords

Personalized PageRank; hybrid graphs; localization; perturbation analysis; random walks on graphs.

Category

Applied Mathematics — Graph Theory, Markov Chains, and Numerical Analysis

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From Single (Symmetry) Reduction via the Optimal System to Double Reduction via Conservation Laws for physically interesting Partial Differential Equations

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Abstract

In this talk, we begin with symmetry classification [1] for a class of nonlinear PDEs using the Lie symmetry method and discuss the fundamental simplifications of variables that arise from the Lie invariance criterion. Following this, we discuss the construction of the optimal system of subalgebras to identify unique group-invariant solutions. It is well known that a double reduction [2] occurs whenever the starting nonlinear PDE possesses a conservation law that is invariant with respect to the symmetry. In the end, a summary of the multi-reduction method [3] will be presented for obtaining invariant solutions of physically interesting PDEs.

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Comparative Lie Symmetry Analysis of Four Fundamental One-Dimensional Wave Equations

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Abstract

A systematic comparative Lie symmetry analysis of four paradigmatic one-dimensional wave equations—the linear wave equation, Sine–Gordon equation, Klein–Gordon-type equation, and Boussinesq equation—is presented. We investigate their symmetry algebras, group-invariant solutions, conservation laws, and integrability properties through unified Lie-theoretic methods. The study reveals how different physical mechanisms, such as dispersion, nonlinearity, and higher-order effects, modify symmetry structures and solution properties.

Keywords

Lie symmetry analysis; one-dimensional wave equations; group-invariant solutions; conservation laws.

Category

ODEs and Dynamical Systems

Modeling Multi-Stage Learning Dynamics Using Piecewise Fractional Differential Equations: A Data-Driven Approach to Personalized Education

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Abstract

This article presents a novel mathematical framework for modelling and optimising personalised learning interventions using piecewise fractional differential equations (FDEs). Recognising that learning progresses through distinct cognitive stages, each characterised by unique memory retention and knowledge integration dynamics, we formulate a multi-stage Caputo-type FDE system in which the fractional order α_i quantifies the memory dependence within each stage. The model parameters are estimated through a large-scale inverse problem solved using the Levenberg–Marquardt algorithm accelerated by fractional adjoint sensitivity analysis and applied to longitudinal performance data collected from eight Zambian universities. Based on the estimated learner-specific parameters, we formulate a fractional optimal control problem to

design adaptive, time-varying interventions. The solution, derived using the Fractional Pontryagin Maximum Principle, reveals two distinct optimal policy regimes: sustained, low-intensity support for high-memory learners ($\alpha > 0.7$) and frequent, high-intensity reinforcement for low-memory learners ($\alpha < 0.5$). A quasi-experimental stepped-wedge implementation demonstrates that the model-derived interventions reduce time-to-mastery by 28–35% and improve final assessment scores by 22% compared with uniform intervention approaches while operating within a constrained budget. This work establishes a rigorous, data-driven bridge between fractional calculus and educational policy, providing a scalable tool for precision education in resource-aware environments.

Keywords

Fractional Differential Equations; Personalised Learning; Optimal Control; Inverse Problems; Educational Data Analytics

Category

Computational Mathematics

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A Numerical Analysis on the Unsteady Flow of a Thermomagnetic Reactive Maxwell Nanofluid over a Stretching/Shrinking Sheet with Ohmic Dissipation and Brownian Motion

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Abstract

The major objective of this investigation is to examine the unsteady flow of a thermomagnetic reactive Maxwell nanofluid over a stretching/shrinking sheet with Ohmic dissipation and Brownian motion. Suitable similarity transformations were used to reduce the governing nonlinear partial differential equations of momentum, energy, and species conservation into a set of coupled ordinary differential equations. The reduced similarity ordinary differential equations were solved numerically using the Spectral Quasi-Linearization Method. The influence of pertinent physical parameters on the velocity, temperature, and concentration distributions was studied

and analysed graphically. Further investigations were carried out on the impact of the Eckert number, Prandtl number, Schmidt number, thermal radiation parameter, Brownian motion parameter, thermophoresis parameter, and chemical reaction parameter on the skin friction coefficient and the surface heat and mass transfer rates. The results were displayed in tabular form. The obtained results reveal that the Maxwell parameter and the unsteadiness parameter reduce the Maxwell nanofluid velocity, while the fluid temperature increases with an increase in the Eckert number and thermal radiation parameter.

Keywords

Maxwell nanofluid; stretching sheet; Ohmic dissipation; Brownian motion; spectral quasi-linearization; Gauss–Lobatto points.

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Fluid Mechanics

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Modelling Delivery-Dependent Course Structures for Blended University Course Timetabling

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Abstract

Scheduling large courses in blended learning modes has become a challenge for universities with growing enrolments and limited physical teaching infrastructure. Existing timetabling approaches rarely account for the different scheduling requirements of face-to-face and online sessions in large courses, where face-to-face sessions are split into smaller groups due to room capacity constraints, whereas online sessions can be delivered as a single synchronized group.

This paper proposes a two-phase hybrid optimization framework that explicitly models delivery-dependent course structures to capture the scheduling requirements of face-to-face and online sessions. The framework begins with an ILP model that generates a feasible initial timetable satisfying all hard constraints. In the second phase, a modality-aware local search procedure refines the timetable using two specialized move operators. Experiments using real data from the University of Dodoma, Tanzania, demonstrate that the proposed approach consistently produces higher-quality timetables than a generic local search while maintaining feasibility.

Keywords

Move operators; blended learning; timetabling problem; large-course scheduling.

Category

Operations Research

MATHEMATICAL BIOLOGY, HEALTH & ENVIRONMENT

Modelling the Impacts of Fishing Nets, Cannibalism and Gamma-Hexachlorocychlohexane on Fisheries in Unreserved-Water Bodies

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Abstract

A mathematical model for the population growth of fish was developed and analysed incorporating natural growth stages of fish, fishing nets and use of gamma-hexachlorocychlohexane (lindane). The zero-fish and fish-existence equilibrium states were established. The effective reproduction rate R_C was derived using the next-generation matrix approach. The analytical analysis revealed that the zero-fish and fish-existence equilibrium states are globally asymptotically stable when $R_C \leq 1$ and $R_C > 1$ respectively. Numerical simulations authenticated the analytical results and further showed that using high-efficiency nets (e.g. gillnets, trawls) together with lindane causes severe declines in freshwater fish populations and can drive some species to extinction. Cannibalism among fish does not cause significant negative effects; instead, it stabilizes the population by curbing over-population. The findings highlight the importance of effective management (e.g. mesh size limits, seasonal closures, protected zones, and removing or tightly controlling the use of lindane) to balance ecological health and human needs

Keywords

Fisheries; Cannibalism; Gamma-hexachlorocychlohexane; Equilibrium; Stability.

Category

Biomathematics

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Analysis of Disease Transmission and Competition Among Multiple Species Using an SIR Epidemic Model with Reaction-Diffusion in a Three-Dimensional Bounded Environment

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Abstract

The transmission of diseases in environments where multiple species interact and compete within a shared ecosystem is still not fully understood. This study proposes and analyzes an SIR epidemic model that includes multiple species coexisting in a common habitat. The model is expressed as a system of nonlinear reaction-diffusion partial differential equations, which describe the spatial and temporal dynamics of various interacting species undergoing infection and recovery processes. This approach extends the traditional SIR epidemic model to an n-species framework within a three-dimensional bounded ecosystem. It also accounts for varying diffusion coefficients, natural mortality rates, birth rates, and inter-species interaction rates. The properties related to the solution are demonstrated using the maximum principle, fixed-point theorems, and the quasi-positivity of the reaction terms. The existence, boundedness, and non-negativity of the solution are established under appropriate initial and boundary conditions. Additionally, we explore the stationary points of the system and derive a threshold parameter, R_0 (Basic Reproduction Number), which determines whether diseases persist or become extinct. The solutions of the proposed model are biologically meaningful, and the well-posedness (existence, uniqueness, positivity, and boundedness) of the solution is confirmed. Numerical simulations illustrate the spatial and temporal spread of infection and support the analytical findings.

Keywords

Epidemic model; Reaction-diffusion; Competition; Coexistence

Category

PDEs and Applications

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Modeling and Global Sensitivity of Bacterial Meningitis Transmission Dynamics in the African Meningitis Belt

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Abstract

The African meningitis belt is a region highly prone to recurrent outbreaks of bacterial meningitis due to its unique climatic and socio-environmental conditions. This study presents a deterministic compartmental model to investigate the transmission dynamics of bacterial meningitis. The model captures the key epidemiological processes governing disease transmission and progression. Fundamental properties of the model, including positivity and boundedness of solutions, are established to ensure its mathematical and epidemiological validity. Furthermore, the disease-free equilibrium is proven to be globally asymptotically stable when the effective reproduction number is less than unity, while the endemic equilibrium exists and is globally asymptotically stable when the reproduction number exceeds unity. A global sensitivity analysis is conducted using Partial Rank Correlation Coefficients (PRCC) to identify the most influential parameters affecting disease transmission dynamics. The results indicate that the transmission rate (β) is the most influential parameter in driving disease spread, while the recovery rate (γ) significantly reduces infection persistence. The findings further show that parameters associated with disease transmission and recovery play significant roles in shaping the dynamics of bacterial meningitis. Overall, these findings provide important insights into the transmission dynamics of bacterial meningitis and may help inform public health strategies aimed at mitigating meningitis outbreaks in high-risk regions.

Keywords

African meningitis belt; Bacterial Meningitis; Basic reproduction number; Global Sensitivity Analysis; Parameter estimation

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Mathematical Epidemiology

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Modeling the Impact of Media Awareness and Farmer Behavioral Response on the Transmission Dynamics and Control of Early Blight Disease in Tomato Plants

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Abstract

Early blight disease caused by *Alternaria solani* significantly affects tomato production worldwide, while excessive fungicide use creates economic and environmental challenges. This study develops a nonlinear mathematical model incorporating media-driven awareness and farmer behavioral responses to improve sustainable disease management. Disease-free and endemic equilibrium points are determined, and the effective reproduction number, R_e, is derived using the next-generation matrix method. Local and global stability analyses of the disease-free equilibrium are performed, while parameter sensitivity is assessed using partial rank correlation coefficient (PRCC) analysis. Numerical simulations in MATLAB show that increased media awareness and farmer adoption of interventions significantly reduce R_e, decrease the transmission rate β_e, and increase the infected removal rate η_B. The results highlight the importance of information dissemination and farmer participation in reducing disease spread and promoting sustainable early blight management.

Keywords

Epidemic dynamics, Effective reproduction number, Transmission rate, Sensitivity analysis, Numerical simulations.

Category

Biomathematics

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Fractional-Order Modelling and Optimal Control of HIV/AIDS and Cryptococcal Meningitis Co-infection with an Explicit Environmental Reservoir

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Abstract

HIV-associated cryptococcal meningitis remains an important cause of morbidity and mortality among people with advanced HIV disease, particularly in resource-constrained settings. This study develops a nonlinear fractional-order model describing the coupled dynamics of HIV infection, progression to AIDS, cryptococcal disease and environmental fungal exposure. The human population is divided into susceptible individuals, HIV-negative individuals with cryptococcal disease, people living with HIV without AIDS, individuals with AIDS, HIV-cryptococcal

co-infected individuals without AIDS and individuals with AIDS-associated cryptococcal disease. An additional state variable represents environmental fungal burden. The model is nondimensionalised before introducing the Atangana-Baleanu-Caputo fractional derivative, thereby preserving the epidemiological interpretation of the model parameters across different fractional orders. Positivity, boundedness and local well-posedness of the solutions are established. The next-generation matrix method is used to derive HIV and cryptococcal invasion quantities and the corresponding overall reproduction number. A binomial observation model links the epidemiological states to monthly cryptococcal-antigen screening data, while profile-likelihood analysis is used to examine parameter identifiability. Three time-dependent interventions representing HIV prevention, cryptococcal-disease management and antiretroviral therapy are incorporated into an optimal-control framework. Numerical simulations demonstrate that the fractional order influences the magnitude, timing and persistence of the epidemiological trajectories, although lower fractional orders do not necessarily imply disease elimination. Among the seven intervention strategies considered, the combined application of HIV prevention, cryptococcal management and antiretroviral therapy produces the greatest reduction in severe disease and environmental fungal burden under the adopted parameter and cost assumptions. These findings support coordinated interventions that simultaneously reduce HIV transmission, improve cryptococcal screening and treatment, and strengthen sustained access to antiretroviral therapy.

Keywords

HIV/AIDS, Cryptococcal Meningitis, Co-infection dynamics, Fractional-order modeling, Atangana-Baleanu Caputo derivative

Category

Mathematical Epidemiology

Effect of Maternally-Derived Immunity on Dengue-Malaria Co-Infection Dynamics

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Abstract

Maternal antibodies provide temporary protection against dengue and malaria in newborns, yet most co-infection models ignore this phenomenon. This study extends a deterministic dengue-malaria co-infection model by incorporating a maternal immunity compartment. Newborns enter this protected class and transition to susceptibility at a rate proportional to antibody waning. All other model structures, including vector compartments and co-infection terms, remain unchanged. The extended model is analyzed for disease-free equilibrium stability. The basic reproduction number R_{0dm} is scaled downward by the fraction $\omega/(\mu_h+\omega)$, where ω is the waning rate. Using Kenyan demographic data (average maternal protection duration 6 months), the effective R_{0dm} reduces from 19.70 to approximately 9.85, yet remains above unity, confirming endemicity. Sensitivity analysis shows that increasing ω (faster waning) raises R_{0dm}, while longer maternal protection delays the average age of first co-infection. These results suggest that childhood vaccination timing should be optimized relative to maternal immunity duration in co-endemic regions.

Keywords

Maternal immunity; Newborn protection; Antibody waning; Co-infection dynamics; Basic reproduction number

Category

Biomathematics

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High-Resolution Spatiotemporal Modeling of NO_2 and NO in Kigali, Rwanda

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Abstract

Background: Combustion-related air pollutants such as nitrogen oxides (NOx) are an increasingly important component of the urban air pollution mixture in rapidly urbanizing sub-Saharan African cities. High-resolution, city-wide spatiotemporal modeling is critical to support health impact assessments, inform data-driven regulatory strategies targeting major emission sources, and guide urban planning efforts aimed at improving air quality and population health.

Material and Methods: We conducted large-scale field measurements and developed high-resolution spatiotemporal models of nitrogen dioxide (NO2) and nitric oxide (NO) in Kigali, Rwanda. Ogawa passive samplers were deployed at 130 locations, including 120 week-long sites to characterize spatial variability and 10 year-long sites to capture seasonal patterns. Measured concentrations were integrated with geographic, meteorological, traffic-related, land-use, and population predictors in linear mixed-effects models to estimate weekly NO2 and NO concentrations at 50 m spatial resolution. Model performance was evaluated using 10-fold cross-validation.

Results: Final models explained 93% and 78% of the spatial variability in NO2 and NO, respectively, with cross-validated R^2 values of 0.85 for NO2 and 0.83 for NO. Predicted concentrations ranged from 1.2–47.5 μg/m^3 for NO2 and approximately 1.0–81.3 μg/m^3 for NO. Annual

mean concentrations were consistently elevated in high-density residential and commercial areas, lower-elevation zones, and near major roadways, underscoring the dominant influence of traffic emissions and urban land-use characteristics. Clear urban-rural contrasts and distinct seasonal and topographic patterns were observed between pollutants.

Conclusions: This study provides the first high-resolution, city-wide spatiotemporal models of NO2 and NO in Kigali. The strong predictive performance and fine spatial resolution enable detailed exposure assessment for epidemiologic studies and provide actionable evidence to support air quality management in rapidly urbanizing African cities. These findings highlight the growing importance of traffic-related emissions in shaping intra-urban exposure gradients in sub-Saharan Africa.

Keywords

Air pollution, Nitrogen dioxide, Spatio-temporal model, Sub-Saharan Africa, Rwanda

Category

Air pollution

Mathematical Modelling of Social Media Influences on Gambling Behaviour: A Sensitivity Analysis

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Abstract

Gambling disorder represents a growing public health concern worldwide, with its prevalence increasingly amplified by the pervasive reach of social media platforms. This study develops and analyses a novel deterministic compartmental model to investigate the dynamic interplay between social media exposure and gambling behaviour. The model structures individuals into susceptible, actively gambling, and quitter compartments, while incorporating two distinct media types: pro-gambling content that drives initiation and anti-gambling content that discourages gambling behaviour. We derive the gambling-free equilibrium and the basic reproduction number R_0, a threshold quantity determining whether gambling behaviour persists ($R_0 > 1$) or dies out ($R_0 < 1$). Local stability analysis confirms that the gambling-free equilibrium is asymptotically stable when $R_0 < 1$. Comprehensive sensitivity analysis using Latin Hypercube Sampling and Partial Rank Correlation Coefficients identifies the influence of pro-gambling media and media-induced quitting rates as the most critical parameters governing gambling dynamics. Media dynamics analysis reveals persistent pro-gambling dominance under baseline conditions, with pro-gambling content reaching substantially higher levels than anti-gambling content. Numerical simulations validate these theoretical findings and illustrate the competing effects of pro- and anti-gambling content. The results provide actionable insights for policymakers and social media platforms: reducing exposure to gambling-promoting content while

simultaneously amplifying recovery-oriented messages could substantially re duce gambling prevalence. This work establishes a theoretical foundation for designing targeted, evidence-based interventions to mitigate gambling related harms in the digital age.

Keywords

Mathematical Modelling, Reproduction number, Social Media, Gambling, Addiction

Category

Mathematical Epidemiology

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A Predictive Model for Optimized Oncolytic Virotherapy with Two Viruses

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Abstract

Cancer is a growing global burden, demanding novel therapies. Combining Vaccinia (VV) and Vesicular Stomatitis (VSV) oncolytic viruses exhibits potent synergy, yet its dynamics and optimal schedule are unclear. I will present a mathematical model that deciphers this "ping-pong" interaction, where VV neutralizes interferon-α to enhance VSV replication. Calibrated with experimental data, our analysis identifies viral infection rates and burst sizes as key efficacy drivers. Crucially, I will show that therapeutic success is maximized by a specific staggered administration: delaying VV by 5-19 hours after VSV, or VSV by 14-24 hours after VV. This work provides a predictive, mathematically grounded protocol to translate synergistic virotherapy into a more effective clinical strategy, contributing to sustainable cancer care.

Keywords

Oncolytic virotherapy, Mathematical modeling, Synergistic therapy, Optimal control theory, Sensitivity analysis, Bifurcation analysis, Vaccinia virus, Vesicular stomatitis virus, Therapeutic optimization

Category

Mathematical Biology

A Mathematical Modeling Approach to Assess Pharmaceutical, Vaccination, and Environmental Clean-Up Interventions for Mpox Transmission Dynamics in the Democratic Republic of Congo

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Abstract

Mpox remains a major public health concern in the Democratic Republic of Congo, where recurrent outbreaks highlight the need for effective and evidence-based control strategies. This study develops a mathematical model to assess the impact of pharmaceutical treatment, vaccination, and environmental clean-up interventions on Mpox transmission dynamics using real epidemiological data from the DRC. A compartmental deterministic model is formulated and calibrated to reported Mpox data, while analytical and numerical methods are used to derive the basic reproduction number and evaluate intervention scenarios. The results suggest that combined control strategies significantly reduce Mpox transmission compared with single interventions. In particular, vaccination lowers population susceptibility, treatment reduces infectiousness and disease duration, while environmental clean-up decreases indirect transmission risks. The study concludes that integrated pharmaceutical, vaccination, and environmental interventions provide a more effective framework for Mpox control and can support public health decision-making in endemic settings such as the DRC.

Keywords

Mpox, mathematical modeling, vaccination, treatment, clean-up intervention, DRC.

Category

Biomathematics

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Modeling the Impact of Housing Structures and Insecticide-Treated Nets on Malaria Vector Control Strategies in Rufiji District, Tanzania

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Abstract

Malaria is a serious and potentially life-threatening disease caused by parasites of the genus Plasmodium. This study assesses the impact of housing structures and Insecticide-Treated Nets (ITNs) on malaria vector control. The vector control model (VCOM) was analysed directly to real data from ACOBISREM (Application of Community Based and Integrated Strategy to Reduce Malaria disease burden) for malaria control in Rufiji District. The study demonstrates that combined interventions are more effective than individual measures in reducing malaria transmission. Sensitivity analysis of the model parameters shows that the biting rate (a) and mosquito death rate $(\overline{\mu_v}, 0)$ are the most sensitive factor driving malaria transmission, and can significantly lower malaria cases. Furthermore, the numerical simulations reverted that combining modifications to house structure with ITNs is more effective in reducing malaria transmission compared to using either intervention alone. These results suggest that a well modified house structure combined with vector strategies such as ITNs, HI, IRS, and spatial repellent to effectively diminish the malaria burden, emphasizing the crucial role of vector management in controlling the disease.

Keywords

Malaria, Model, Data, ITNs, VCOM, ACOBISREM, EIR

Category

Mathematical Epidemiology

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Bridging the Treatment Gap: Modelling the Impact of ART Access for Immigrants on HIV/AIDS Transmission in Botswana

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Abstract

The Human Immunodeficiency virus (HIV) remains a major public health concern in Botswana, with significant strides made in treatment coverage among citizens. However, for many years, the immigrant population (which comprised about 7% of the population in 2016) was excluded from accessing antiretroviral therapy (ART), raising concerns about their role in sustaining HIV transmission. This study formulates and analyses a deterministic compartmental model that incorporates both citizens and immigrants, stratifying infected individuals by treatment and viral suppression status. The model includes key features such as ART adherence, viral suppression dynamics, and Acquired Immunodeficiency Syndrome (AIDS) progression. A core component of the model is the introduction of a policy intervention that grants ART access to immigrants, as implemented by the Botswana government in 2019. The model steady states are determined and analysed in terms of the basic reproduction number, (R_0^{pre}). It is observed that the model has two unique steady states, the disease-free steady state and an endemic equilibrium point. The disease-free equilibrium point is globally asymptotically stable if $R_0^{\text{pre}} < 1$, while the endemic equilibrium point is globally asymptotically stable if $R_0^{\text{pre}} > 1$. Comparing pre-policy and post-policy scenarios, we find that the reproduction number is significantly higher before

the policy change, indicating that excluding immigrants from treatment increases overall HIV transmission. Numerical simulations are carried out to determine the impact of the immigrant population on the incidence of HIV/AIDS dynamics. Our results suggest that extending ART access to immigrants significantly reduces the overall HIV burden and can accelerate progress towards epidemic control.

Keywords

Modelling, HIV/AIDS treatment, Policy, Immigrants.

Category

Biomathematics

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Pneumonia Modelling Dynamics with Sensitivity Analysis Using Normalized and PAWN Methods

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Abstract

Pneumonia is a global health challenge that causes significant human mortality annually, particularly among children and the elderly. This study developed a mathematical model to investigate the transmission dynamics of the disease, incorporating treatment interventions targeting infected groups. The analysis of model solutions and the stability of both disease-free and endemic equilibrium points were conducted using the Jacobian matrix and Lyapunov functions.

The effective reproduction number (R_e) was computed using the Castillo-Chavez method, while sensitivity analysis was performed using normalized sensitivity indices and the PAWN method. The findings reveal that pneumonia-infected populations exhibit ($R_e > 1$), indicating persistence of the disease, whereas disease-free states demonstrate ($R_e < 1$), suggesting both local and global asymptotic stability. Locally, the normalized sensitivity index method was used to identify the most influential parameters around the equilibrium point. Globally, the PAWN method was employed to assess the influence of all parameters in the model. The natural death rate (μ) was found to be the most influential parameter, with its cumulative distribution function (CDF) showing high dispersion, followed by the transmission rate (β). The study further suggests that the PAWN method is an essential approach, as it effectively identifies the range of parameter influence across the entire system. This insight is crucial for the effective control of pneumonia spread, which requires proper management of key parameters alongside the provision of adequate treatment for infected individuals to significantly reduce disease prevalence. These findings highlight the important roles of health workers and policymakers in implementing effective intervention strategies.

Keywords

Pneumonia; Basic reproduction number; Normalized index; PAWN sensitivity analysis

Category

Mathematical Epidemiology

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Southern African HIV/AIDS Dynamics Analysis Integrating Statistical and Mathematical Modelling

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Abstract

HIV is a fatal infectious disease spread mainly by sexual transmission recently; that is highly prevalent in endemic areas of the world. A simple subset of an epidemic population model for HIV-AIDS can be split into the following: Susceptible, HIV-Infected and AIDS-infected, (SIA). We propose and use a nonlinear mathematical model to analyse the dynamics of HIV transmission in a population. The equilibrium points for the model are computed where the stability of the disease-free equilibrium is governed by the basic reproductive number. PRCC technique is utilized to investigate the sensitivity analysis of the model. Numerical simulations are computed to analyse the qualitative behaviour of the model compartments, using MATLAB ode45 solver. Optimal control analysis is introduced where the optimal problem is formulated using Pontryagin's Maximum Principle. The simulations were carried out using the forward-backward sweep method, allowing us to determine the impact of the control strategies on each of the model compartments, particularly the infectious class. The influence of a single control and the combination of both is investigated. The comparison of the two strategies in eliminating the reproductive number is done by simulation of contour plots. It is found that the use of more than one control is the most optimal when compared to the use of a single control.

Keywords

Adherence, PrEP

Category

Biomathematics

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Effect of Human Behavior and Fund Cut on the Transmission Dynamics of HIV/AIDS Through Fractional Order Mathematical Model

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Abstract

Human Immunodeficiency Virus (HIV) remains a major public health challenge, particularly in Sub-Saharan Africa, despite progress in antiretroviral therapy (ART), testing, and prevention programs that have reduced AIDS-related deaths and new infections. These achievements are increasingly threatened by behavioral challenges such as inconsistent condom use, poor treatment adherence, stigma, and declining HIV testing, compounded by reductions in global health funding. Funding cuts may disrupt treatment access, weaken prevention strategies, and reduce awareness efforts. This study develops a fractional-order HIV transmission model to assess the combined impact of behavioral change and funding reductions on HIV/AIDS dynamics. Fractional-order models are preferred because they capture memory and hereditary effects, including past adherence, behavioral patterns, and long-term prevention exposure [1, 2, 4, 3]. The model extends the Susceptible–Infected–Treated framework to inform resilient HIV intervention policies.

Keywords

HIV; AIDS; SIT; Fractional-order model

Category

Biomathematics

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A Mathematical Dynamics of Drug-Sensitive and Drug-Resistant Strains of Tuberculosis Disease Transmission with Parameter Uncertainties Analysis in Tanzania

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Abstract

Tuberculosis (TB) remains a major global health challenge and causes significant mortality. This study developed a mathematical model to analyze TB transmission in Tanzania (2018–2023), incorporating both drug-sensitive and drug-resistant strains, treatment interventions, and parameter uncertainties. An effective reproductive number R_e is determined; consequently, a stability analysis of the model solutions at both disease-free and endemic equilibrium points is conducted using the Jacobian matrix and Lyapunov functions. Analytically, TB-infected populations exhibit $R_e > 1$, confirming global stability, whereas disease-free states demonstrate $R_e < 1$, suggesting local and global asymptotic stability. The study findings showed that parameter uncertainties significantly affected disease prevalence and control, delaying reduction beyond 50 months. Increasing treatment rates reduced R_e from 2.5 to 0.6, although high uncertainty (40–80%) slowed convergence. Overall, this study provides a useful framework for understanding TB dynamics and guiding effective control strategies for policymakers and health practitioners.

Keywords

Tuberculosis; Drug-sensitive; Drug-resistant; Parameter Uncertainties

Category

Mathematical Epidemiology

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A Caputo Fractional-Order Mathematical Model and Optimal Control Analysis of Lethal Yellowing Disease in Coconut Palms

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Abstract

Lethal Yellowing Disease (LYD) is a major threat to coconut palm production, causing substantial economic losses in many coconut-growing regions. This study developed and analysed a fractional-order mathematical model using the Caputo derivative to investigate LYD transmission dynamics while incorporating memory effects. The model's positivity, boundedness, equilibrium points, and basic reproduction number (R_0) were analysed, followed by sensitivity analysis, numerical simulations, optimal control, and cost-effectiveness analysis. The results showed that higher transmission rates, vector recruitment, and carrying capacity increased disease spread, whereas increased vector mortality reduced infection. Numerical simulations indicated that the fractional-order model predicted slower disease progression than the classical model. Combined control strategies including planting of resistant coconut palms, rogueing of infected palms, and vector control measures were the most effective in reducing disease prevalence. Cost-effectiveness analysis identified vector control effort alone as the most economical

intervention. Overall, the proposed model provides a useful framework for understanding LYD transmission and supporting effective disease management.

Keywords

Lethal Yellowing Disease; Caputo fractional-order model; Memory effect; Hyers–Ulam stability; Pontryagin's Maximum Principle.

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Mathematical Modelling and Optimal Control of Antimicrobial Resistant-Typhoid Fever Incorporating Public Health Education

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Abstract

Typhoid fever remains a major public health challenge in many developing regions, and its management is further complicated by the growing threat of antimicrobial resistance (AMR). This study develops and analyses a deterministic two-strain model to investigate the transmission dynamics of antimicrobial-resistant typhoid fever. The model incorporates time-dependent interventions, including public health education, sanitation improvement, and treatment. Analytical results show that the model admits disease-free, mutant-dominant, and coexistence equilibria, and that disease elimination is achievable when the associated reproduction numbers are maintained below unity. Global sensitivity analysis using the Elementary Effects method identifies public health education and the ingestion rate of contaminated food or water as the most influential factors driving AMR transmission. Numerical simulations under constant control reveal that higher mutation rates do not necessarily increase resistance, whereas inadequate public health education accelerates its spread. Comprehensive optimal control simulations highlight the importance of each intervention (sanitation, treatment, and public health education) in reducing typhoid fever transmission. Furthermore, the results show that integrated interventions provide the greatest reduction in disease burden, with public health education playing a pivotal role. In addition, cost-effectiveness analyses reveal that the combination of sanitation and treatment is the most cost-effective strategy under limited resources. The study highlights the critical role of sustained public health education and integrated intervention strategies in mitigating the emergence and spread of AMR in typhoid fever.

Keywords

Typhoid fever; Sensitivity analysis; Control strategies; Antimicrobial resistance; Reproduction number.

Category

Mathematical Modelling

Mortality Attributable to Short-Term Exposure to Outdoor Air Pollution

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Abstract

Air pollution is a major global public health concern, contributing to millions of premature deaths annually. Among the key pollutants, particulate matter with an aerodynamic diameter of less than 2.5 micrometres ($PM_{2.5}$) is one of the most significant contributors to adverse health outcomes. However, its health impacts remain insufficiently quantified in low- and middle-income countries, particularly in sub-Saharan Africa. This study investigates the short-term association between $PM_{2.5}$ exposure and all-cause mortality using a Generalized Additive Model. The results are quantified in terms of relative risk, attributable number, and attributable rate, computed at both monthly and annual scales.

Keywords

Particulate matter; air pollution; mortality; generalized additive model; relative risk; attributable number.

Category

Mathematical Epidemiology

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Mathematical Analysis of Seasonal Malaria Transmission with Integrated Control Strategies

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Abstract

Persistent transmission of Malaria remains a major challenge in endemic settings, despite multiple control efforts. This study develops a modified SEIRS–SEI host–vector model that incorporates exposed classes, seasonal variation in mosquito recruitment, and time-dependent control measures, including insecticide-treated nets, indoor residual spraying, treatment, and environmental management. Optimal control theory is applied to determine strategies that minimize both disease burden and intervention costs. Analysis of the basic reproduction number (R0) and numerical simulations show that coordinated application of interventions, particularly when aligned with seasonal peaks, substantially reduces transmission [1]. Sensitivity analysis highlights the mosquito biting rate and intervention coverage as critical factors influencing disease spread[2, 3]. The findings indicate that isolated interventions are insufficient, whereas a combined and optimally timed strategy yields better outcomes. This model offers a practical quantitative tool for designing efficient and targeted malaria control policies in endemic regions.

Keywords

Malaria; Reproduction Number; Sensitivity; Stability; Optimal Control

Category

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Graph-theoretic Approach for R_0 and R_e in Continuous-Time and Fractional-Order Epidemiological Models

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Abstract

The computation of the basic reproduction number (R_0) and the effective reproduction number (R_e) plays a central role in understanding disease transmission dynamics and control strategies in epidemiological modeling. These quantities serve as fundamental threshold parameters indicating whether an infectious disease dies out or persists in a population. However, for multi-compartment and high-dimensional epidemic models, the classical next-generation matrix approach becomes analytically cumbersome and computationally expensive due to matrix inversion and eigenvalue computations.

To address this challenge, we proposed a graph-theoretic approach for computing R_0 and R_e in continuous-time and fractional-order epidemiological models. The method transforms the infectious subsystem into a weighted directed multigraph, where nodes (vertices) represent infected compartments and directed edges describe transmission and transition pathways. Using systematic graph reduction techniques, reproduction numbers are obtained without explicit matrix inversion or eigenvalue computation, significantly reducing analytical and computational complexity in high-dimensional systems. Importantly, the method preserves the classical threshold property while improving the interpretability of dominant infection pathways and subsystem interactions.

Several epidemiological examples in Sylas and Mureithi (2026) demonstrate the effectiveness and applicability of the approach, showing that it provides a robust and efficient alternative to classical next-generation matrix techniques for complex epidemic systems.

Keywords

Trivial node; self loop; weighted graph; multigraph; irreducible matrix; strongly connected.

Category

Biomathematics

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A Note on the Fractional-Order Model of Malaria Disease Transmission in Border Regions: A Case Study of South Africa and Mozambique

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Abstract

Malaria remains a severe public health challenge in sub-Saharan Africa, where cross-border human and vector migration, economic constraints, environmental variables, and seasonality significantly hinder elimination efforts. This study investigates the dynamics of malaria transmission across the border region of South Africa and Mozambique using a mathematical modelling approach.

A novel non-linear, two-patch fractional-order model in the Atangana-Baleanu-Caputo sense is proposed, explicitly incorporating both host and vector mobility. Theoretical properties of the model, including positivity and boundedness of solutions, are established to ensure ecological validity. Fixed-point techniques are applied to prove the existence and uniqueness of the model's solutions, while the next-generation matrix method is used to derive the patch-specific basic reproduction number R_0. A conditionally stable fractional Euler approximation scheme is developed for numerical analysis, and global sensitivity analysis using partial rank correlation coefficients (PRCC) identifies the key transmission drivers.

Analytical results show that the disease-free equilibrium is locally asymptotically stable when $R_0 < 1$. Sensitivity analysis indicates that human and vector birth rates, mosquito biting rates, disease transmission probabilities, and progression rates from exposed to infectious classes positively influence malaria transmission, whereas vector mortality and human recovery rates have a negative effect. Numerical simulations further demonstrate that memory effects inherent in the fractional-order system accelerate convergence towards the disease-free equilibrium. The

model is validated using empirical malaria case data from Mozambique, showing a robust statistical fit.

The findings reveal that seasonal fluctuations strongly influence human migration and cross-border malaria transmission. The study concludes that regional malaria elimination requires coordinated cross-border interventions, particularly in vector control and synchronized management of imported malaria cases.

Keywords

Malaria disease; fractional-order model; human and vector migration; sensitivity analysis; seasonality; South Africa; Mozambique.

Category

Biomathematics

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Network Curvature Reveals Climate-Sensitive Transmission Pathways for Influenza Control in High School Contact Networks

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Abstract

Respiratory infectious diseases, such as influenza, are highly sensitive to climate variability, with seasonal changes in temperature and humidity influencing transmission dynamics, particularly in school settings where close contact is frequent. Understanding how climate-sensitive transmission interacts with contact network structure is essential for improving preparedness and intervention strategies. This study investigates how Forman–Ricci curvature, a geometric network measure, can identify structural vulnerability and improve control of influenza-like illness in a high school contact network.

Using high-resolution face-to-face interaction data from the SocioPatterns project, we constructed a weighted network representing student contacts and computed curvature alongside traditional centrality measures. We then simulated influenza transmission using a curvature-modulated network-based susceptible–infected–recovered model, incorporating transmission scenarios consistent with climate-sensitive respiratory disease dynamics, including increased transmissibility under conditions favourable to viral survival.

Our results show that negatively curved edges identify critical bridges connecting student groups, which act as climate-sensitive transmission pathways and accelerate outbreak spread. Epidemic simulations demonstrate that increased transmissibility, consistent with favourable climatic conditions, leads to earlier and larger epidemic peaks. Importantly, intervention strategies targeting curvature-identified structural bridges significantly reduce epidemic size and delay the peak of infections compared with conventional strategies targeting highly connected individuals.

These findings underscore the importance of accounting for both climate-sensitive transmission and contact network geometry when designing intervention strategies for respiratory diseases in schools. Network curvature provides a novel tool for identifying high-risk transmission pathways and improving targeted interventions. This approach can support climate-informed public health planning and strengthen resilience against climate-driven respiratory disease outbreaks in educational settings and beyond.

Keywords

Forman–Ricci Curvature; Network-Based Epidemic Modelling; Climate-Sensitive Transmission; Influenza Dynamics; Contact Networks

Category

Biomathematics

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Mathematical Modelling of Malaria Integrating Temperature, Rainfall, and Vegetation Index

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Abstract

Environmental factors such as temperature, rainfall, and vegetation index play a crucial role in the transmission dynamics of malaria. Accurately quantifying the complex relationships between these variables and the malaria burden poses a significant challenge. In this study, we developed a host-mosquito mathematical model to investigate the impact of temperature, rainfall, and normalized difference vegetation index (NDVI) on malaria transmission dynamics calibrated with the Burundi case study. Mathematical analysis explored the equilibria, stability, and computation of the model's threshold values. Numerical simulations suggest that temperature, rainfall, and vegetation index affect the transmission dynamics of malaria. Temperature and NDVI appear to exhibit a more pronounced influence among these factors. The conditions conducive to malaria transmission include a mean monthly temperature range of [20–25°C], and an averaged monthly NDVI range of approximately [0.4–0.6]. The reproduction number was used as a quantitative measure to predict the impact of temperature, rainfall, and NDVI on malaria transmission dynamics across Burundi. The results suggest a progressive increase in the reproduction number over time, suggesting a threat of the rising number of cases in Burundi if drastic control measures are not implemented.

Keywords

Mathematical model; Malaria; Reproduction number; Temperature; Rainfall; NDVI.

Category

Biomathematics

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A predator-prey inspired model for corruption dynamics with feedback from law enforcement and community engagement

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Abstract

Corruption remains a persistent socio-economic challenge that undermines institutional integrity, governance, and sustainable development. The study analyses a nonlinear dynamical system to examine the impact of law enforcement and community engagement on corruption dynamics. Inspired by predator–prey interactions, the model conceptualizes corruption as a self-reinforcing process moderated by enforcement mechanisms and societal participation. The influence corruption number governing persistence or elimination is derived using a graph-theoretic approach, highlighting the importance of enforcement intensity and sustained community engagement. Results indicate that enforcement alone is insufficient for long-term corruption control, whereas integrated strategies combining strong institutional action with active public participation are more effective in reducing corruption levels. The findings provide theoretical insights into institutional and societal responses to corruption and offer a quantitative framework for evaluating anti-corruption policies and promoting transparency, accountability, and sustainable governance.

Keywords

Corruption dynamics; Nonlinear dynamical system; Law enforcement; Community engagement; Graph-theoretic approach

Category

Mathematical Epidemiology

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AI, DATA SCIENCE, STATISTICS & FINANCE

Development of a Hybrid Osprey–Bayesian Optimisation Framework for Enhancing Explainability in AI-Driven Distance Learning Systems

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Abstract

Advances in artificial intelligence continue to expand opportunities for improving learning analytics, early intervention, and pedagogical decision-making in distance education. Ensuring that these systems remain transparent and trustworthy, however, requires optimisation techniques grounded in rigorous mathematical principles. This study proposes an enhanced Explainable Artificial Intelligence (XAI) framework that addresses the instability and sensitivity of the widely used Local Interpretable Model-Agnostic Explanations (LIME) technique. A hybrid optimisation strategy integrating the Enhanced Osprey Optimisation Algorithm (EOOA) with Bayesian Optimisation (BO) is introduced to refine LIME's key hyperparameters. EOOA ensures a balanced exploration–exploitation search, while BO provides probabilistic guidance for efficient convergence. The resulting system improves the stability, fidelity, and interpretability of explanations in real-time learning analytics. The optimised framework will be evaluated on fidelity, stability, comprehensibility, coverage, and timeliness, using interaction and performance data from Ladoke Akintola University, Ogbomoso, Nigeria distance learning platform. The study will show how mathematical optimisation and educational research can collaboratively strengthen sustainable, transparent digital learning systems.

Keywords

Hybrid optimisation; explainable artificial intelligence; metaheuristic algorithms; distance learning analytics; model interpretability.

Category

Optimisation

Modelling Meteorological Drought in Tanzania Using Machine Learning Models Integrated with the Standardized Precipitation Index

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Abstract

Meteorological drought is a major climate hazard affecting water resources, agriculture, ecosystems, and livelihoods in Tanzania. This study develops an Artificial Neural Network (ANN) framework to model meteorological drought using the Standardized Precipitation Index (SPI) integrated with ERA5 meteorological and climate variables. An ANN (7–7–1) was trained using eleven predictors, including wind speed, mean sea-level pressure, relative humidity, soil temperature, and aerosol optical depth, with 1-, 2-, 3-, and 6-month lags for SPI3 and SPI6 prediction.

Results showed substantial interannual drought variability, with mild drought occurring most frequently (21.5–28.9%), while severe and extreme droughts are less common. The best ANN model achieved high predictive accuracy, with testing R^2 values of 0.9592 for SPI3 and 0.9382 for SPI6. These findings demonstrate that drought evolution is strongly influenced by local atmospheric conditions and large-scale climate processes. The proposed framework provides a reliable tool for drought forecasting, early warning, water-resource management, agricultural planning, and climate adaptation in Tanzania.

Keywords

Meteorological drought; artificial neural network; machine learning; Standardized Precipitation Index; ERA5; Tanzania.

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Climatology

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Data Science and AI-Driven Models for Optimizing Renewable Energy Systems in Developing Economies

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Abstract

Sustainable development requires a global shift to renewable energy sources, especially in developing countries where reliable power supplies and universal access to electricity remain significant challenges. This paper proposes a novel approach for optimizing wind and solar energy systems using data science and artificial intelligence (AI). Neural networks and long short-term memory (LSTM) models are employed to improve energy output forecasting and distribution efficiency. The proposed prediction framework incorporates real-time data on solar irradiation, wind speed, and grid dynamics.

The approach combines supervised learning with quantitative optimization while accounting for the economic and infrastructural constraints of Eastern Africa. It balances electricity supply and demand to reduce energy waste and improve system stability. Case studies based on local data demonstrate improved prediction accuracy and reduced operational costs. This study highlights the importance of collaboration among mathematicians, data scientists, and policymakers in developing scalable AI-driven solutions for enhancing energy accessibility and sustainability, thereby contributing to the achievement of the Sustainable Development Goals in resource-constrained settings.

Keywords

Renewable energy; artificial intelligence; data science; sustainable development; energy optimization.

Category

Data Science and Mathematics for AI

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Do Proxy Measures of Social Capital Align with Complex Network Properties? Insights from Systematic Review, Formal Derivation, and Empirical Validation

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Abstract

Social capital is a broad concept that plays a fundamental role in understanding social systems and their applications. This study presents a systematic review of English-language publications from the 1950s to 2023 to identify quantifiable aspects of social capital and their corresponding complex-network measures. Following the PRISMA 2020 framework with a modified PICO approach, 522 records were screened across six major scholarly databases and grey literature sources, resulting in 167 studies that met the inclusion criteria.

The review produced a structured three-dimensional model comprising social relations, social contagion, and civic engagement, within which sixteen operationalized social-capital attributes were identified. A second corpus of 72 studies was synthesized to establish a crosswalk between these attributes and complex-network metrics. Formal analysis demonstrated consistent alignment with small-world and heavy-tailed network regimes reported in the literature. These findings were further validated through mathematical derivations and simulations on canonical network models, illustrating how measurable aspects of social capital correspond to small-world and scale-free network structures.

The resulting framework provides a transparent approach for analysing social dynamics using complex-network theory and establishes a foundation for future empirical investigations of social capital.

Keywords

Complex networks; social capital; sociometry; computational social science.

Category

Graph Theory

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Reductions of the SPC Model to Classical Small-World and Scale-Free Network Models

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Abstract

This study presents the theoretical validation of the Small-World Preferential and Community (SPC) model through formal reductions to classical network generative models. Using reduction theory, the study establishes structural and behavioural equivalences between the SPC model and widely used network models. The findings demonstrate that the Watts–Strogatz (WS), Newman–Watts–Strogatz (NWS), Barabási–Albert (BA), and Holme–Kim (HK) models emerge as special cases of the SPC model under specific parameter constraints. These reductions confirm the theoretical soundness, flexibility, and generality of the SPC model across multiple network structures. The results further show that the SPC model can effectively represent diverse complex network domains without being restricted to a single network formation pattern.

Keywords

Small-world networks; scale-free networks; network modelling; community structure; complex networks.

Category

Graph Theory

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A Portfolio Choice Problem When Mortality Risk Is Age-Dependent

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Abstract

This paper extends the risk capacity constraint framework to incorporate age-dependent mortality risk. While the existing literature models the retiree's death time as exponentially distributed (see Chen et al. 2020), this assumption implies a constant hazard rate that is inconsistent with actuarial evidence. We replace the exponential distribution with a Gompertz–Makeham mortality law, under which the hazard rate increases with age, and study the resulting optimal portfolio choice problem under a risk capacity constraint. The time-varying hazard rate destroys the reduction to an infinite-horizon problem, and the value function satisfies a parabolic partial differential equation in wealth and time. We establish the existence of a time-dependent wealth threshold, $W^*(t)$, separating the constrained and unconstrained investment regions. The optimal investment strategy depends jointly on wealth and age, and the wealth-cycle property of the original model no longer holds.

Keywords

Age-dependent mortality risk; Gompertz–Makeham mortality law; time-varying hazard rate; unconstrained investment regions; optimal investment strategy.

Category

Financial Mathematics

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Generalized Growth Curve Model and Its Extensions: From Estimation to Hypothesis Testing

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Abstract

The Generalized Growth Curve Model (GGCM) [1] extends the classical growth curve model [2] to tensor-valued data, enabling the study of multi-way multivariate observations with complex dependency structures. This paper provides a comprehensive review of the methodological developments in GGCM, beginning with the foundational work on parameter estimation under

separable and doubly separable covariance structures [1, 3]. We then examine recent advances in hypothesis testing, including the formulation and testing of general trilinear hypotheses via the likelihood ratio test [4]. A major extension to the GGCM framework is the Tensor Mixed Effects (TME) model [5], which incorporates both fixed and random effects to handle multilevel variabilities inherent in tensor data. We present the TME model's theoretical foundations, including maximum likelihood estimation via a double flip-flop algorithm, existence conditions, and identifiability results. The theoretical foundations, computational algorithms, and practical considerations are synthesized, and directions for future research are identified.

Keywords

Generalized Growth Curve Model; Likelihood Ratio Test; Separable Covariance; Tensor Data; Trilinear Hypothesis.

Category

Mathematical Statistics.

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Mathematics for Governable Early-Warning Systems: Data-Efficient, Long-Memory and Risk-Aware Decision Models for East African Public Infrastructure

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Abstract East African public institutions and industries increasingly need data-driven early-warning systems for health planning, mobile-money risk monitoring, climate and agricultural services, revenue administration, education, and digital public infrastructure. Yet these settings often face scarce data, noisy administrative records, delayed reporting, limited computing resources, long-memory effects, andconstrained institutional capacity. This presentation develops a mathematical framework for governable early-warning systems under such conditions. The framework combines data-efficient learning, long-memory dynamics, stochastic risk modelling, optimization, and governance gates for evaluating whether a model can be maintained, secured, contested, and safely discontinued. The framework is illustrated through East African use cases, including transaction-risk early warning, health-systemprioritization, climate and agricultural advisory risk, and public-sector AI readiness. The central argument is that mathematics for sustainable development should develop reliable, interpretable, and governable decision systems under real African constraints.

Keywords

Data-efficient learning; long-memory dynamics; governable AI; early-warning systems; public infrastructure.

Category

Mathematics for AI

Modelling of University Academic Progression: Application of Multistate Markov Models

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Abstract

University academic performance is an important tool to assess the quality of education. This study used the Multistate Markov to model undergraduate university academic progression at a university in Lusaka, Zambia.

A retrospective cohort study was conducted of which a total of 347 students were followed up from 2017 to 2021. The flexible parametric Royston-Parmar survival model was used to predict overall probability of transition from the state to state.

The transition probability to higher stages increased with time, showing substantial levels of academic progression. The probability of ever reaching year 4 was low due to most students failing to pass all courses especially by the end of second year among other factors.

The study recommends that prospective mixed-methods studies are needed to explore the academic, social, and institutional factors affecting student progression in these critical states.

Keywords

Academic progression; Multistate Markov model; Transition probability.

Category

Applied Statistics

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A Modelling Framework for Parameter Estimation and Optimal Control of STH–Malaria Co-infection Dynamics

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Abstract

Soil-transmitted helminths (STH) and malaria are among the parasitic diseases that pose significant public health challenges in endemic regions, where their co-endemicity results in co-infection that can complicate control efforts. The World Health Organization 2030 agenda calls for modelling contributions toward eliminating STH as a public health problem and reducing malaria incidence by 90%. However, parameter scarcity often hinders such efforts. In this study, we develop a deterministic model of STH–malaria co-infection, estimate key parameters using Markov Chain Monte Carlo (MCMC) methods, and extend the model to an optimal control framework for cost-effectiveness evaluation. Synthetic data were generated under Gaussian noise. The next-generation matrix was used to compute the basic reproduction number, and Pontryagin's theory was applied to evaluate four intervention strategies (malaria prevention, STH prevention, treatment, and environmental decontamination), while the incremental cost-effectiveness ratio (ICER) was employed to assess their efficiency. Numerical simulations were performed in MATLAB. The results show that increasing the number of data points and reducing noise substantially decrease parameter uncertainty. Under the formulated model, the MCMC algorithm estimated six key parameters, including the transmission rate and co-infection parameters. The model confirms that integrating prevention and treatment strategies (Strategy II) provides the most economically viable approach, with an ICER of 0.00026. Overall, this study highlights the importance of sufficient data for reliable parameter estimation in

guiding optimal control strategies for STH–malaria co-infection management. However, the analysis is based on synthetic data and assumed intervention costs; future work should incorporate real-world data and explore alternative noise structures in parameter estimation.

Keywords

Soil-transmitted helminth; Malaria; Mathematical model; Optimal control theory; Control strategies; Parameter estimation.

Category

Biomathematics

Forecasting Water Level in Mtera Dam Using Machine Learning Techniques

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Abstract

This study aimed to forecast water levels in Mtera dam using statistical and machine learning models. Six models (ARIMA, ETS, XGBoost, GRU, LSTM and CNN-LSTM) were used in forecasting water level in Mtera dam. Five metrics were used to measure the performance of all models. LSTM outperformed other models by having good values in metrics (Low values in MAE, MSE, RMSE, MAPE and high value in R^2). For that case LSTM model was used in forecasting water level for five years period. Future research may incorporate more parameters like rainfall, temperature, humidity, and outflow discharge to make the model more robust. Integrating refined models with advanced sensors could pave the way for real-time dam water level forecasting, enhancing preparedness and response mechanisms.

Keywords

Water Level; Forecasting; Machine Learning

Category

Data Science

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Derivation of a Risk Model that Incorporates Inhomogeneous Claim Counts and Financial Risks

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Abstract

In the insurance industry, ruin or insolvency occurs when an insurer's total liabilities exceed its assets, rendering it unable to meet future claims. Traditional ruin probability models, such as the Cramer-Lundberg model, assume a homogeneous Poisson claim process with a constant arrival rate and exclude financial risk factors that influences an insurer's solvency, which may not accurately reflect real-world market dynamics. In practice, claim arrivals are often in-homogeneous due to seasonal variations, policyholder behaviors, and financial economic factors. This study develops an extended risk model that incorporates in-homogeneous claim counts and financial risks, offering a more realistic framework for estimating ruin probabilities in non-life insurance. The proposed model is derived using integral-differential equations and Cramer-Lundberg approximations, providing a refined approach to risk assessment. The findings offer valuable insights for insurers and regulators in designing risk management strategies to mitigate insolvency risks and ensure financial stability.

Keywords

Risk model; Classical risk process; Inhomogeneous claims; Financial risk; Cramér–Lundberg model

Category

Financial/Insurance Mathematics

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A Stochastic Control Framework for Liquidity Crises in Investment Funds with Regime Switching and Jump Risks

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Collaborators and Affiliations

None.

Abstract

Liquidity crises pose significant challenges to investment funds due to sudden investor redemptions, market volatility, and changing economic conditions. This study develops a regime-switching jump-diffusion model with optimal control to examine liquidity risk dynamics and determine effective liquidity management strategies for investment funds. The model incorporates stochastic asset returns, compound Poisson subscription and redemption processes, stochastic short-term interest rates, and Markov regime switching between normal and stressed market conditions. The liquidity management problem is formulated as a stochastic optimal control problem and solved through the associated HJB equation using numerical techniques, including finite-difference methods, policy iteration, and Monte Carlo simulation. Model parameters are estimated from daily Net Asset Value (NAV), outstanding units, and short-term interest-rate data. The results indicate that redemption intensity, market volatility, and regime persistence are the primary drivers of liquidity stress. The study concludes that regime-sensitive optimal liquidity policies significantly enhance fund resilience and reduce liquidity-crisis risk under adverse market conditions.

Keywords

Liquidity crises; investment funds; Markov regime switching; jump-diffusion dynamics; Hamilton–Jacobi–Bellman equation.

Category

Financial Mathematics

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The Effect of Random Seed on Risk of Type I or II Error During Multiple Imputation of Binary Outcomes Missing at Random

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Abstract

Missing data remain a major challenge in epidemiological research. Multiple imputation (MI) is widely used for handling data assumed to be missing at random (MAR), but because MI relies on stochastic draws from predictive distributions, results may vary across random seeds and numbers of imputations, potentially affecting statistical inference. This study evaluated the stability of MI under varying random seeds and numbers of imputations and compared its performance with inverse probability weighting (IPW) and machine learning methods at different levels of missingness.

We simulated data based on a two-stage population-based epilepsy survey conducted in two urban informal settlements in Nairobi, Kenya. Residents were first screened for epilepsy using a standardized tool, after which screen-positive individuals underwent neurological assessment and diagnosis by a trained neurologist. The binary outcome was epilepsy diagnosis (1 = epilepsy, 0 = no epilepsy). Outcome missingness was imposed at 20%, 30%, and 50% under a MAR mechanism conditional on study site. MI was implemented using logistic regression with three random seeds (1, 100, and 1000) and varying numbers of imputations (5, 50, and 100). Logistic regression analyses identified factors associated with epilepsy diagnosis, including site, education, marital status, employment, age, and sex. Results were compared with IPW and a random forest classifier. Performance was evaluated using point estimates, standard errors, statistical significance, bias, and the area under the receiver operating characteristic curve (AUC).

In the complete dataset, only site and age were statistically significant predictors of epilepsy diagnosis. Under MI, results varied across random seeds, with employment becoming significant while age lost significance in some scenarios. Bias increased with higher missingness, raising the risk of Type I and Type II errors, although estimates became more stable as the number of imputations increased. IPW produced results comparable to MI but with larger standard errors at higher missingness levels. The random forest model showed superior predictive performance, particularly at higher missingness, achieving an AUC of 0.84 compared with 0.75 for MI, while producing lower bias than IPW. MI, however, generated smaller standard errors.

The study concludes that MI may produce unstable inferential results as missingness increases, with random seeds and numbers of imputations influencing statistical significance and bias. While MI and IPW performed similarly at lower missingness levels, the random forest approach demonstrated greater robustness and predictive accuracy under substantial missing data. These

findings highlight the importance of carefully evaluating missing-data methods in epidemiological research and suggest that machine learning approaches may complement conventional imputation techniques.

Keywords

Multiple imputation; Machine learning; Inverse probability weighting; Missing data.

Category

Mathematical Statistics

A hybrid multilevel and machine learning models for student dropout prediction

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Abstract

Student dropout poses a significant challenge to education systems worldwide, with profound consequences for individuals and society. Accurate early identification of at-risk students is essential for designing effective interventions. Educational data are inherently hierarchical, students are nested within households, schools, or classrooms, violating the independence assumption of standard machine learning (ML) models and potentially leading to biased predictions. While multilevel statistical models have long addressed this issue for explanatory purposes, systematic evidence on integrating hybrid multilevel ML models with conventional single-level algorithms for predictive tasks remains limited. This study presents a unified hybrid framework that integrates multilevel random intercepts with three diverse ML families, gradient boosting machines (XGBoost), multilayer perceptrons (MLP), and stacking ensembles, to predict student dropout while explicitly accounting for household-level clustering. Using simulated and real-world hierarchical educational data, we benchmark nine modeling configurations, comparing standard implementations against their multilevel counterparts with random intercepts. Results demonstrate that incorporating a multilevel random intercept consistently improves predictive performance across all ML families, with the largest gains observed for XGBoost and MLP.

Keywords

Student dropout prediction; Multilevel machine learning; Hierarchical educational data; Random effects.

Category

Mathematical Statistics

Misclassification Probabilities for Linear and Quadratic Classifiers in Repeated-Measures Data: A Systematic Review

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Abstract

Despite extensive work on misclassification probabilities for linear classifiers in repeated-measures settings using Edgeworth-type expansions, a comprehensive systematic review covering both linear and quadratic discriminant analysis (LDA and QDA) for repeated-measures data remains to be developed. This review synthesizes classical theory and recent developments, including moment-based Edgeworth expansions for quadratic classifiers. Key findings indicate that, while LDA misclassification probabilities have been extensively studied, the literature on QDA, despite its theoretical advantage for heterogeneous covariances in longitudinal data, is less developed. Recent advances, including Edgeworth approximations for both linear and quadratic classifiers, have improved error-rate estimation; however, persistent challenges remain regarding small-sample accuracy, computational feasibility, and generalizability across repeated-measures designs. This paper consolidates existing knowledge, identifies critical gaps, and outlines future research directions for robust misclassification probability estimation in both linear and quadratic classification of repeated-measures data.

Keywords

Quadratic Discriminant Analysis (QDA); Misclassification Probability; Repeated-Measures Data; Growth Curve Models; Edgeworth Expansion

Category

Multivariate Statistics (MSC 62H30)

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Application of K-Means Clustering to Mental Health Profiles of University Students in Northern Tanzania

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Collaborators and Affiliations

Abstract

Mental health challenges among university students are a growing concern worldwide, yet evidence from Tanzania remains limited. This study investigated the prevalence and patterns of mental health issues among 250 students from four universities. Data were collected through online questionnaires covering demographics, academic indicators, and standardized measures, including the Depression Anxiety Stress Scales (DASS-21), Pittsburgh Sleep Quality Index (PSQI), and Perceived Stress Scale (PSS). K-means clustering, together with the Elbow Method and Silhouette Analysis, was employed to identify three distinct student profiles: mentally healthy, at-risk, and distressed. More than 60% of students fell into the at-risk or distressed categories, highlighting widespread vulnerability. Financial support was significantly associated with cluster membership ($p < 0.05$), while demographic and academic variables showed

no significant effects. The findings underscore the urgent need for tailored interventions, such as targeted counseling, screening, and expanded financial aid, while providing evidence to inform university policies and enhance student well-being in Tanzania.

Keywords

K-means clustering; mental health profiles; university students; standardized measures.

Category

Statistics

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Misclassification Probability Approximation for a Quadratic Classifier with Repeated Measurements

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Abstract

In classification, any decision rule involves a risk of misclassification. We study this problem for two populations with unequal covariance matrices and repeated measurements. A quadratic classification rule is derived for the growth curve model, and the unknown parameters are estimated. Using these estimators, we compute the moments of the quadratic classifier and employ an Edgeworth-type expansion to approximate the misclassification probabilities. Numerical simulations compare the probabilities obtained using true and estimated mean parameters, as well as those computed via Monte Carlo simulations. The results show that misclassification decreases as the separation between the two groups grows, demonstrating improved classification accuracy for well-separated populations.

Keywords

Approximation; Misclassification probability; Repeated measurements; Quadratic classifier.

Category

Mathematical Statistics

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A One-Factor Short Rate Model Driven by Skew Diffusions

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Abstract

Interest rates do not revert to a single long-run equilibrium. Instead, they fluctuate across monetary policy cycles, reverting toward structurally different equilibrium levels depending on prevailing economic conditions. Classical one-factor short-rate models, such as those of Vasicek

and Cox–Ingersoll–Ross, assume a single equilibrium level and therefore fail to adequately capture yield curve dynamics during regime transitions.

This study introduces a one-factor short-rate model that accommodates multiple mean-reversion levels through the use of skew diffusions. The proposed process behaves as a standard diffusion away from prescribed interest-rate levels but experiences asymmetric drift corrections at those levels, causing the short rate to be attracted toward or repelled from specific thresholds depending on whether they represent support or resistance levels.

A closed-form bond pricing formula is derived, and the resulting bond market is shown to be free of arbitrage. The proposed model provides a tractable and economically meaningful alternative to traditional multi-factor models for pricing fixed-income securities in markets characterized by regime-dependent mean reversion.

Keywords

Zero-coupon bond pricing; multiple mean reversion; local time; no-arbitrage; interest rate derivatives.

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Financial/Insurance Mathematics

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From Global to Local Dependence: Recent Advances Beyond Pearson's Correlation

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Abstract

Pearson's correlation coefficient remains the most widely used measure of statistical dependence and provides a complete characterization in the Gaussian setting. However, it is well known to perform poorly in the presence of nonlinear dependence, heavy-tailed distributions, and time-varying dependence structures, where it may lead to misleading conclusions. Motivated by these limitations, a growing literature has proposed alternative measures that capture more complex forms of dependence. A particularly flexible approach is local Gaussian correlation (LGC), which describes dependence by locally approximating the joint distribution with a Gaussian model, allowing dependence to vary across the sample space. This contribution highlights recent advances in LGC-based methodology, with particular emphasis on a block bootstrap test for comparing dependence structures across time periods. By combining LGC with stationary bootstrap techniques and data-adaptive grid selection, the proposed framework enables robust inference for nonlinear dependence in serially dependent and heavy-tailed time series, as illustrated in applications to energy and financial markets.

Keywords

Statistical dependence; nonlinear and local dependence measures; local Gaussian correlation

Category

Statistics

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Estimation of Threshold Effect of Kenyan Shilling Exchange Rate on Rwandan Economy: A Semiparametric Model Approach

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Abstract

This study investigates the impact of fluctuations in the Kenyan shilling exchange rate on Rwanda's economy, focusing on identifying threshold effects on trade and economic growth. The research addresses limitations of traditional linear regression models by applying a semi-parametric threshold model, which better captures nonlinear relationships between exchange rates and economic indicators. Using annual data from 1996 to 2018, the study analyzes imports, exports, and gross domestic product (GDP).

Findings reveal significant threshold levels of the Kenyan shilling exchange rate at 7.78 for imports, 7.52 for exports, and 7.39 for GDP in Rwandan francs. Beyond these thresholds, changes in the exchange rate have a significant positive effect on Rwanda's economic growth, influencing trade flows and GDP performance. The results confirm the importance of nonlinear threshold effects in understanding exchange rate dynamics. The study concludes that monetary authorities, particularly the National Bank of Rwanda, should carefully monitor exchange rate movements to maintain competitiveness and avoid excessive appreciation of the Rwandan franc.

Keywords

Nonparametric regression estimation; Semiparametric estimation; Threshold effect.

Category

Mathematical Statistics

Triadic Closure and Self-Organised Growth in an Early National Twitter Network: Evidence from Tanzania

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Abstract

Understanding the growth of online social networks remains a fundamental challenge in network science, particularly for networks from underrepresented regions. This study investigates an early Twitter (now X) network centred on Tanzania, comprising 1,514,435 accounts and 61,175,824 follower–following links collected during 2018–2019, to examine whether its structure is consistent with a self-organised growth mechanism based on local recommendation.

An extended friend-of-a-friend model is employed in which a new user first connects to a randomly selected account and subsequently links to neighbouring accounts with probability q.

The empirical network is compared with the model using several structural properties, including the in-degree distribution, clustering structure, assortative mixing, average path length, and eigenvector centrality.

The results show that the Tanzanian Twitter network exhibits an in-degree distribution with an exponent close to 2, weak degree assortativity, short path lengths, and a clustering profile that decreases approximately monotonically with degree. These characteristics are accurately reproduced by the friend-of-a-friend model with $q = 1$ and $n_q = 40$, corresponding to the observed mean degree. Unlike previously studied Twitter networks from countries such as the USA and Japan, the Tanzanian network appears to be driven primarily by local triadic closure rather than the influence of large commercial actors. The findings provide evidence that early-stage national social networks exhibit structural signatures of self-organised growth and demonstrate the importance of regional network data for validating mathematical models of social connectivity.

Keywords

Online social networks; triadic closure; Twitter; network growth; sub-Saharan Africa.

Category

Network Science

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MATHEMATICS EDUCATION & SUSTAINABLE DEVELOPMENT

Building Sustainable Higher Education and Industrial Engagement in Applied Mathematics: The Ethiopian-Norwegian Network in Computational Mathematics (ENNCoMat)

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Abstract

The Ethiopian higher education sector has undergone rapid expansion, growing from only a handful of public institutions to about 50 within recent decades. While this growth has increased access, the domestic development of specialized fields such as engineering and advanced industry remains heavily constrained by the underrepresentation of STEM subjects and limited industrial research capacity. To address these challenges, Hawassa University and the Norwegian University of Science and Technology (NTNU) have sustained a fruitful 15-year collaboration aimed at establishing robust postgraduate programmes in applied mathematics and statistics. This abstract presents the methodologies and broader impacts of the *Ethiopian-Norwegian Network in Computational Mathematics* (ENNCoMat), a strategic institutional initiative that expands this historical framework to include Adama Science and Technology University and Arba Minch University, alongside Norway's Nord University.

Funded to improve the quality of advanced mathematics education, ENNCoMat contributes directly to regional sustainable development goals by combining curriculum co-development, student mobility, gender inclusion, and industrial engagement. To establish sustainable educational infrastructure, the project focuses on updating and co-creating open-source MSc and PhD courses, such as *Advanced Numerical Methods*, *Linear Methods*, and *Optimisation*. By distributing source code and lecture materials openly on GitHub, the network ensures that pedagogical resources remain freely accessible and locally adaptable, thereby overcoming structural and economic textbook barriers.

Crucially, the partnership transforms theoretical mathematics education into a practical catalyst for domestic development. Adopting frameworks pioneered by the European Consortium for Mathematics in Industry (ECMI), the project organizes Modelling Weeks and Ethiopian Study Groups with Industry (ESGI). In these interactive forums, European and African researchers work alongside postgraduate exchange students to apply computational techniques to authentic problems posed by local businesses and public organizations. Furthermore, the network facilitates intensive programming summer camps designed to bridge technical skills gaps and encourage female undergraduate students to pursue postgraduate studies in mathematics. To date, the network has successfully supported the exchange of ten MSc and four PhD researchers. Ultimately, ENNCoMat serves as a scalable model for how North–South institutional collaborations can build independent mathematical research capacity to drive long-term sustainable development.

Keywords

Computational mathematics; educational collaboration; study groups with industry; capacity building; North–South partnership.

Category

Industrial Mathematics / Mathematics Education

References

Effects of Digital Mathematics Instructional Materials on Secondary School Students' Conceptual Understanding and Procedural Fluency

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Abstract

The shortage of mathematics teachers in Tanzania's secondary schools remains a critical challenge affecting students' learning and achievement in mathematics. In response to this challenge, Digital Mathematics Instructional Materials (DiMIMa), which are curriculum-aligned audio-visual mathematics lessons, were designed and developed as an intervention that supports students' self-learning of mathematics during the lesson in contexts where regular guidance is limited due to teacher shortages. Therefore, this study examined the effects of DiMIMa video lessons on secondary school students' conceptual understanding and procedural fluency in mathematics. A quasi-experimental design was employed, involving 454 Form One and Three students from control and experimental schools. The intervention focused on two mathematics topics, Algebra and the Earth as a Sphere, respectively, for the two classes. Pre- and post-test results were used to compare students' performance using independent and paired sample t-tests. The findings showed that, in procedural fluency, Form One students in the experimental school performed significantly better than those in their control group ($p = 0.014$, $d = 0.32$), whereas in Form Three the post-test difference was not significant ($p = 0.536$, $d = 0.09$). However, the experimental group showed substantial within-group gains, with $d = 0.50$ in Form One and $d = 0.93$ in Form Three. In conceptual understanding, post-test differences between the groups were not statistically significant in both classes, $p = 0.89$ and $p = 0.100$, respectively, for Forms One and Three. However, paired-samples results showed significant improvement within-group among students who used DiMIMa video lessons, with moderate improvement in Form One $d = 0.50$ and large gain in Form Three $d = 0.87$. The study concludes that DiMIMa video lessons can support students' self-learning in mathematics and improve their conceptual understanding and procedural fluency, particularly in contexts affected by mathematics teacher shortages. Therefore, the study recommends integrating DiMIMa video lessons into secondary school mathematics instruction as a complementary strategy to support students' self-learning, particularly in resource-constrained environments.

Keywords

DiMIMa Video Lessons, Mathematics self-learning, Conceptual Understanding, Procedural Fluency, Teacher Shortages, Secondary Schools

Category

Mathematics Education

References

Integration of Ethnomathematics with Problem-Based Learning: Approaches and Issues

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Abstract Growing interest in integrating ethnomathematics with Problem-Based Learning (PBL) highlights the importance of connecting mathematics to students' cultural experiences and lived realities. However, its systematic integration into formal school mathematics remains limited in many African contexts, including Uganda, and there is less focus on its overcome. The goal of the study was to develop a framework for integrating ethnomathematical activities into PBL to enhance cultural appreciation and mathematical literacy while identifying implementation challenges. Using a qualitative ethnographic approach, data were collected through document analysis, field observations, artifact analysis, and semi-structured interviews with cultural practitioners and educators from selected Ugandan communities. Findings revealed that Ugandan cultural practices, including terracing, threshing floor design, embroidery patterns, traditional storage systems, monument construction, and the local game Omweso, embody rich mathematical concepts such as algebra, geometry, optimization, and spatial reasoning. These practices were transformed into PBL activities, assignments, and inquiry-based projects. In conclusion, teachers may consider some of the identified cultural activities by integrating them into their instructional practices through problem-based learning.

Keywords

Problem-based learning; ethnomathematics; indigenous knowledge; didactic inference; secondary mathematics; integration.

Category

Mathematics Education

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Mathematics Modelling Perspectives for Sustainability Learning in Ugandan Lower Secondary Schools: A Transformational Approach

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Abstract

Mathematical modelling and sustainable learning reinforce the teaching of lower secondary mathematics within Uganda's Competence-Based Curriculum (CBC). The CBC integrates cross-cutting issues intended to promote knowledge application and behavioural change among learners. Grounded in transformational learning theory, the study focused on three modelling perspectives: social-critical, contextual, and realistic/applied. It investigated three secondary school mathematics teachers in Wakiso District, Kampala, Uganda. As part of the intervention, teachers were introduced to mathematical modelling within the transformational learning framework, while the researcher acted as a participant observer, facilitating and monitoring the process. Findings revealed that the contextual perspective served as a bridge, connecting abstract concepts to familiar realities through Ugandan stories, such as land disputes and brokers. The practical perspective empowered students as researchers and critical thinkers, linking mathematics to national development and personal survival. Teachers also integrated affective and values-based domains, fostering empathy, reflection, and value-driven peer guidance.

Keywords

Mathematical modelling; Competence-Based Curriculum; sustainable learning.

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Classroom Dialogue in Large Mathematics Classes: Integrating Participatory Lectures, Think–Pair–Share, and Whole-Class Discussion in a Secondary School Classroom in Tanzania

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Abstract

Large class sizes remain a persistent challenge in mathematics education, particularly where curriculum demands, limited resources, and overcrowded classrooms constrain learner-centred pedagogy. While interactive approaches are widely advocated, there is limited observation based evidence on how classroom dialogue and interaction patterns change when teachers integrate feasible strategies in large classes. This study examined how participatory lectures, think–pair–share, and whole-class discussion shaped classroom dialogue, interaction patterns, and student engagement in Tanzanian secondary schools. Guided by constructivist learning theory, the study adopted a mixed methods approach within a design-based research framework. Data were collected through classroom observations, interviews, and focus group discussions, and analysed using descriptive statistics and thematic analysis. Findings indicate a shift from teacher-centred instruction to more dialogic practices, with increased interaction, participation, and peer-based formative feedback. Despite time and classroom management challenges, structured strategies supported more dialogic teaching without reducing class size.

Keywords

Large mathematics classes; classroom dialogue; participatory lecture; think–pair–share; whole class discussion; constructivist pedagogy

Category

Mathematics Education

References

Trends in Mathematics Education Research in Tanzania: A Bibliometric Analysis

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Abstract

This paper analyses trends in mathematics education research in Tanzania, focusing on thematic areas over time, levels of education, and research approaches. The paper presents a bibliometric analysis of dissertations and journal articles obtained from institutional research repositories. Furthermore, publications from various research databases, including SCOPUS, AJOL, and other Tanzanian sources, were analysed. The generated themes were analysed using Microsoft Excel by counting frequencies and comparing themes over time.

The analysis indicates a strong focus on instructional practices, with teaching and learning, assessment, and technology integration featuring prominently. Most of the research focused on ordinary-level secondary education, followed by primary school mathematics. Advanced-level secondary school mathematics attracted comparatively fewer research studies. An increasing research focus at the undergraduate level, in both mathematics and mathematics education, is also evident. Research in this field needs to be strengthened through enhanced long-term follow-up studies and the strengthening of postgraduate coursework programmes.

Keywords

Mathematics education; Research trends; Bibliometric analysis; Educational research; School mathematics.

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Clustering Students on Their Experience of Learning Undergraduate Mathematics in Tanzania

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Abstract

This paper examines clusters of students based on their performance and experiences in the teaching and learning of undergraduate mathematics. The study involved 406 undergraduate mathematics students from different bachelor's degree programmes in Tanzania. A mixed-method research approach was employed using questionnaires and interviews for generating data. Thematic analysis and K-means cluster analysis were used for analysing the data.

The study revealed both systemic and student-based factors leading to low performance in undergraduate mathematics courses. Furthermore, three clusters were generated from the data based on performance and gender sensitivity. The three clusters (low-performing positive students, positive higher-performing students, and average-performing gender-sensitive students) were similar in how they characterised lecturers, teaching, and the nature of mathematics courses. Cluster membership was dependent on the year of study and degree programme. The paper recommends initiatives directed towards students, as well as improvements in university teaching timetables, transportation, and degree programme structure.

Keywords

Undergraduate mathematics; Mathematics performance; Students' experience; Cluster analysis; Learning mathematics.

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Reinforcement Learning for Personalized STEM Learning: A Dynamic Adaptive Framework

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Abstract

Personalized learning in STEM education remains constrained by static student models and single-objective optimization strategies. This paper proposes a Reinforcement Learning-Driven Adaptive Learning Framework (RL-ALF) that dynamically personalizes learning pathways by integrating Deep Knowledge Tracing with a Deep Q-Network policy engine. The framework models instruction as a Markov Decision Process, where an intelligent tutoring agent selects actions — concept presentation, practice exercises, or scaffolding — based on continuously updated learner knowledge states. A multi-objective reward function simultaneously optimizes concept mastery, engagement, cognitive load, and learning efficiency. Evaluation metrics include learning gain, knowledge mastery, and recommendation accuracy. The proposed architecture addresses key limitations in existing adaptive systems, including data efficiency, real-world scalability, and multi-objective pedagogical coordination. This work contributes a scalable, pedagogically informed RL framework with potential to significantly improve individualized STEM instruction.

Keywords

Reinforcement learning; Personalised learning; STEM education; Intelligent tutoring systems; Knowledge tracing

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The Relationship Between Calculations Involving Negative Numbers and the Mental Number Line in Zambia

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Abstract

This study examines the relationship between students' performance in addition and subtraction involving negative integers and the development of the Mental Number Line (MNL) among Grade 8 students in Zambia, grounded in a developmental psychology perspective. According to Dehaene (1997), humans possess an innate "number sense," which enables the representation of numerical magnitude along a mental continuum known as the MNL. The development of this representation is fundamental to understanding numerical order and arithmetic operations.

Results indicate that while students showed high accuracy in calculations involving only positive integers, performance declined significantly when negative integers were included. A consistent pattern of errors was observed, particularly the overgeneralization of natural number rules to negative number operations ("natural number bias"), suggesting that difficulties are conceptual rather than purely procedural.

Analysis of MNL tasks revealed that accuracy decreased as the numerical range increased and that students had greater difficulty locating negative numbers than positive ones. In addition, substantial individual differences in MNL development were identified using Percent Absolute Error (PAE).

Correlation analyses showed a moderate negative relationship between PAE and calculation scores, indicating that more accurate MNL representations are associated with better performance. Conversely, a positive correlation was found between PAE and typical errors related to natural number bias.

Keywords

Negative numbers; mental number line; typical errors; addition and subtraction.

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Mathematics Education

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